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OPTIMIZATION OF THE LAYOUTS OF STEEL PILES USING OPTIMALITY CRITERIA

by

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Prepared under the direction of Professor Kevin Z. Truman

A thesis presented to the Sever Institute of

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ABSTRACT

OPTIMIZATION OF THE LAYOUTS OF STEEL PILES USING OPTIMALITY CRITERIA

by Alan Hoback

ADVISOR: Professor Kevin Z. Truman

May, 1991

Saint Louis, Missouri

The layout of steel piles is optimized to find the least weight structure. The pile cross-sections and the pile orientations (batters) are varied using optimality criteria. The handling of topological variables by the optimality criteria approach is thoroughly discussed.

Several examples are given. One example is a retaining wall for which a two-dimensional pile layout can be used to represent a segment of the wall. Another example is a control tower. Four other examples are dam structures.

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OPTIMIZATION OF THE LAYOUTS OF STEEL PILES USING OPTIMALITY CRITERIA

1. INTRODUCTION

Optimization techniques have come under greater use as the demand has grown for least cost structures. Modern structural optimization began to be developed in the 1960's (1,2,3). Two categories of optimization methods exist: analytic and numerical methods (4,5). Analytic methods can lead to the exact optimal solution, but these methods require all of the optimization parameters to be expressed explicitly in terms of the optimization variables. Numerical methods are used for structural optimization problems. They are used because the problem does not need to be explicitly expressed. Numerical methods usually involve iterative processes which alter an initial design towards an optimal design. Each optimization method has advantages and disadvantages. The optimization method must be selected after considering the characteristics of the problem to be solved.

The layout of piles is to be optimized. The objective of the optimization is to find the least weight of steel HP-14 sections that can be used to satisfy the demands of the loading conditions.

Previous work has been performed in the area of pile optimization. A program was developed in 1981 by Hill which attempts to optimize the layouts of piles (6). To reduce the weight of steel a pile deletion process was used. The optimization procedure consisted of first finding the optimal pile slopes, then finding the optimal pile spacing within specified zones, and deleting piles until the stresses and displacements are near their limits. The piles are deleted in an iterative process by eliminating the most and/or the least stressed piles.

An optimality criteria approach was used in this study. The pile optimization problem is nonlinear. The design limits such as stresses change nonlinearly with respect to the variables. Frequently problems are linearized by finding the gradients with respect to the variables of the weight and the constraints.

The optimality criteria approach is developed from the nature of an optimal design point. Optimality criteria methods were first used in the 1970's and have been applied to a variety of structural optimization problems. A paper by Venkayya presents the optimality criteria method based on a strain energy criterion (7). The method is developed to solve optimization problems with a large number of variables.

The method quickly arrives at an optimal design while using a relatively small amount of computational effort. Venkayya applies the method to truss and frame optimization by allowing member sizes to vary.

The optimization methods which are developed are applied to an existing pile group analysis computer program. See Appendix 1 for further information about the analysis methods.

2. MATHEMATICAL FORMULATION OF THE METHOD

2.1 STATEMENT OF THE FORM OF THE OBJECTIVE AND CONSTRAINTS

The objective function of the optimization process is the function to be minimized. The goal of the pile optimization process is to find the minimum weight of steel that can be used to satisfy the given constraints. The objective function is:

$$W_T = \sum_{i=1}^{N} \rho_i V_i \tag{2.1}$$

where W_T is the total weight, ρ_i is the density, and V_i is the volume for the structural element i, and N is the number of piles.

Structures are subject to a variety of constraints. Some of the constraints are: member stress limits, displacement limits, soil capacity, minimum and maximum member sizes, and the layout restrictions. The constraints generally place an upper or lower bound on the value of a parameter such as stress. An example of a stress constraint is:

$$h_j = \sigma_j - \overline{\sigma}_j \leq 0 \qquad j=1,\ldots,m_1 \tag{2.2}$$

where h_j is the value of the constraint j, σ_j is the stress, $\overline{\sigma}_j$ is the upper bound of the stress in element j, and m_1 is the number of stress constraints. The constraint is satisfied when h_j is zero or negative. To improve the numerical conditioning of the constraints, Equation 2.2 may be rewritten as:

$$h_j = \frac{\sigma_j}{\bar{\sigma}_j} - 1 \leq 0 \qquad j=1,\ldots,m_1 \tag{2.3}$$

A constraint on the lower bound of a parameter such as member size can be written as:

$$h_j = \frac{-I_{xx\,j}}{\underline{I}_{xx\,j}} + 1 \leq 0 \qquad j=1,\ldots,m_2 \tag{2.4}$$

where $\underline{I}_{xx\,j}$ is the lower bound of the moment of inertia $I_{xx\,j}$ for member j, and m_2 is the number of lower limit size constraints.

2.2 OPTIMALITY CRITERIA FORMULATION

The problem statement is:

minimize:

$$W_T = \sum_{i=1}^{n} \rho_i V_i \tag{2.5}$$

subject to:

$$h_j \leq 0 \qquad j=1,\ldots m \tag{2.6}$$

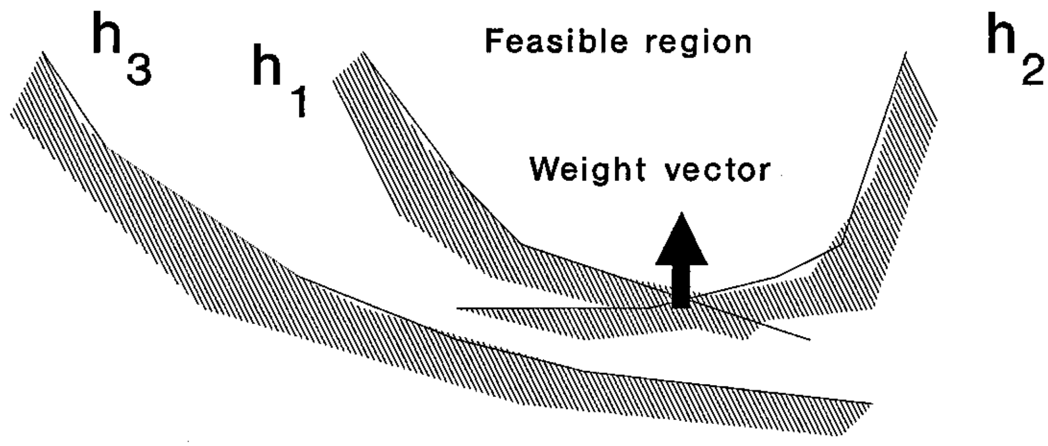
where m is the total number of constraints.

Satisfaction of the KUHN-TUCKER conditions is necessary for a point in design space to be a local minimum of the given problem (8). One optimal point exists when a problem is convex. If the design space is convex then the satisfaction of the KUHN-TUCKER conditions is sufficient for a design to be the global optimum. The pile orientation optimization problem is non-convex. A method that can be used to find the global optimum is the usage of multiple starting points for the optimization process. Forming the KUHN-TUCKER conditions requires the Lagrangian which is:

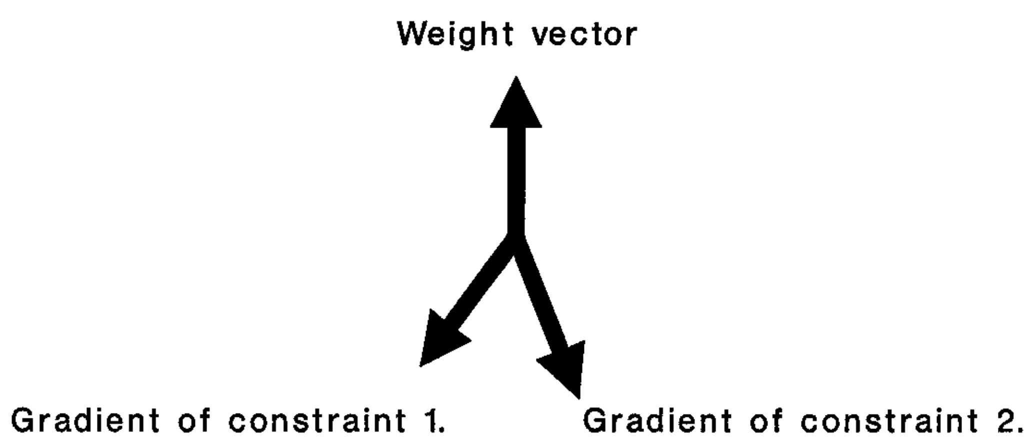
$$L = W_T + \sum_{j=1}^{m} \lambda_j h_j \qquad (2.7)$$

where λ_j is the Lagrange Multiplier for the constraint j.

The first necessary condition for a local minimum dictates that a further improvement in the weight is prevented by constraints. See Figure 2-1. Constraints are active when $h_j=0$, and passive if $h_j<0$. The constraints which are active at the optimum design point will form a vector opposing the weight gradient. In Figure 2-1 constraints 1 and 2 are active. To perform a vector addition of weight and constraint gradients a scaling variable is required to account for the units of the gradients. The scaling is performed by the Lagrange Multiplier. The formula for the vector addition of the gradients is the derivative of the Lagrangian with respect



(a)



(b)

Figure 2-1 Gradient Vectors

(a): Constrained space, (b): Gradients.

to the design variables which can be written as:

$$\frac{\partial W_T}{\partial d_i} + \sum_{j=1}^{m} \lambda_j \frac{\partial h_j}{\partial d_i} = 0 \qquad i=1,\ldots,n \tag{2.8}$$

where there are n design variables d_i. This is the first necessary condition. Note that λ_j does not vary at the optimum.

The other necessary conditions are found by looking at active and passive constraints. A constraint gradient is used in the vector sum to oppose improvements in weight only when the constraint is active. This requires that λ_j is zero when h_j is not zero, and λ_j is not equal zero when h_j is zero.

$$\lambda_j h_j = 0 \qquad j=1,\ldots,m \tag{2.9}$$

The final conditions dictate that the Lagrange Multipliers must be positive and the constraint must not be violated.

$$\lambda_j \geq 0 \qquad j=1,\ldots,m \tag{2.10}$$

$$h_j \leq 0 \qquad j=1,\ldots,m \tag{2.11}$$

Selecting the correct set of constraints as active is the most complicated portion of the optimization process. Equation 2.8 may be rewritten to provide the optimality criteria:

$$-\left(\sum_{j=1}^{m} \lambda_j \frac{\partial h_j}{\partial d_i}\right) \Big/ \frac{\partial W_T}{\partial d_i} = 1 \qquad i=1,\ldots,n \tag{2.12}$$

This criteria must be satisfied for the n variables.

The optimality criteria for the i^{th} variable is used as the efficiency of the variable. The i^{th} variable (d_i) is altered by the value of the i^{th} optimality criteria in the following recurrence formula:

$$d_i^{v+1} = d_i^v \left(-\left(\sum_{j=1}^{m} \lambda_j \frac{\partial h_j}{\partial d_i} \right) / \frac{\partial W_T}{\partial d_i} \right)^{\frac{1}{r}} \qquad i=1,\ldots,n \qquad \textbf{(2.13)}$$

where v is the index of the iteration number and r is the convergence control parameter.

The optimality criteria is less than one when the component of the objective or weight gradient for the i^{th} variable is greater than the resistance provided. When the weight gradient is positive, the variable is free to decrease by an amount roughly proportional to the value of the optimality criteria. If the resistance is stronger than the weight gradient then the optimality criteria is greater than one and the variable should be increased. Taking the r^{th} root of the optimality criteria where $r>1$ assures that the prediction of d_i for the next iteration does not greatly overshoot the optimum value. A reasonable value for the convergence control parameter (r) is taken as 2.

Rewriting Equation 2.13:

$$d_i^{v+1} = d_i^v \left(1 + \left(\left(-\left(\sum_{j=1}^{m} \lambda_j \frac{\partial h_j}{\partial d_i} \right) / \frac{\partial W_T}{\partial d_i} \right) - 1 \right) \right)^{\frac{1}{r}} \qquad \textbf{(2.14)}$$

and expanding it using the binomial theorem results in a linear recurrence equation:

$$d_i^{v+1} = d_i^{v}\left(1+\frac{1}{r}\left(\left(-\left(\sum_{j=1}^{m}\lambda_j\frac{\partial h_j}{\partial d_i}\right)\Big/\frac{\partial W_T}{\partial d_i}\right)-1\right)\right) \tag{2.15}$$

The values of the Lagrange Multipliers to be used in the variable recurrence relation must be estimated. Estimation of the multipliers may be performed using either a recurrence relationship or by solving a set of linear equations. A recurrence relation method requires assuming the initial values of the Lagrange Multipliers. Another disadvantage is that the process is slow to converge. The second method which was the method used was a linear equation solution method. This method requires that the constraints must be known a priori to be contained in either the active or passive set of constraints.

A formula for the change in the active constraints is used to produce the linear equation solution method. The equation for the change and linear approximations of the change in the j^{th} active constraint can be written as:

$$\Delta h_j = h_j(d_i+\Delta d_i) - h_j(d_i) = \sum_{i=1}^{n}\frac{\partial h_j}{\partial d_i}\Delta d_i \qquad j=1,\ldots,m \tag{2.16}$$

where $h_j(d_i)$ is the value of the constraint given the variable d_i. The expected value of the active constraints after any iteration should approach zero, therefore the previous value of the constraint should be equal to the negative of the change.

$$-h_j(d_i) = \sum_{i=1}^{n} \frac{\partial h_j}{\partial d_i} \Delta d_i \qquad j=1,\ldots,m \tag{2.17}$$

The change in the design variables can be found using the variable recurrence equation which contains the first KUHN-TUCKER necessary condition. Substituting the variable change:

$$\Delta d_i^v = d_i^{v+1} - d_i^v \tag{2.18}$$

and Equation 2.15 into the constraint change equation yields:

$$-h_j(d_i) = \sum_{i=1}^{n} \frac{\partial h_j}{\partial d_i}\left(\frac{1}{r}\left(\left(-\sum_{s=1}^{m} \lambda_s \frac{\partial h_s}{\partial d_i} \Big/ \frac{\partial W_T}{\partial d_i}\right)-1\right)\right) d_i^v \qquad j=1,\ldots m \tag{2.19}$$

and then rearranging yields:

$$r\,h_j(d_i) = \sum_{s=1}^{m} \lambda_s \sum_{i=1}^{n} \left(\frac{\partial h_j}{\partial d_i}\frac{\partial h_s}{\partial d_i} \Big/ \frac{\partial W_T}{\partial d_i}\right) d_i + \sum_{i=1}^{n} \frac{\partial h_j}{\partial d_i} d_i \qquad j=1,\ldots,m \tag{2.20}$$

Passive variables are variables which are forced to maintain a certain value. A cross-section size can be held at the maximum or minimum value by making it passive. An additional term is required to account for the change in the constraints by the applied change in the passive variables.

$$r\,h_j(d_i) = \sum_{s=1}^{m} \lambda_s \sum_{i=1}^{n_1} \left(\frac{\partial h_j}{\partial d_i} \frac{\partial h_s}{\partial d_i} \Big/ \frac{\partial W_T}{\partial d_i} \right) d_i + \sum_{i=1}^{n_1} \frac{\partial h_j}{\partial d_i} d_i + r \sum_{i=n_1+1}^{n} \frac{\partial h_j}{\partial d_i} \Delta d_i$$

$$j = 1, \ldots, m$$

(2.21)

where n_1 is the number of active variables.

2.3 SELECTING THE ACTIVE CONSTRAINT SET

The solution method consists of assuming an active set of constraints and forming the linear equations. If the solution of the equations satisfies the KUHN-TUCKER conditions (Equations 2.8-2.11) then the solution is the optimum for the current iteration. The variable recurrence equations can then be solved.

The process of selection of the active Lagrange Multipliers can be performed by first choosing the constraints which are near violation as active. The set of linear equations is formed for the active constraints. The set is altered by causing newly violated constraints to be considered active and removing constraints with negative Lagrange Multipliers. To aide in approaching a beneficial set it may be decided that at any trial only the most violated constraints will enter the solution. When beginning a new optimization iteration a good initial selection is the final set of active constraints from the previous optimization iteration.

If a feasible set of active constraints can not be found then an extension of the search method is used. It is possible to grow a set. First only one constraint is given consideration. It is considered the only constraint applied to the design. After a set of active constraints is found then the number of constraints applied to the design is incremented. The new set of active constraints is found beginning with the previous set. This process is continued until all of the constraints given are applied to the design.

The process of choosing active constraints could use refinement. The selection process could be solved more directly by solving a second order problem. The linear equations have the first KUHN-TUCKER condition embedded in them. The equations to be solved would be the linear equations with the remaining KUHN-TUCKER conditions. This is a second order problem since either h or λ must be zero. A numerical method for solving the set of second order equations relies on linearization. The linearization results in a very poorly conditioned problem.

3. APPLICATION OF OPTIMIZATION METHOD

The pile layouts were analyzed by an altered version of the US Army Corps of Engineers program: Pile Group Analysis (CPGA) Computer Program (9). The optimization process requires the constraint gradients which are the changes in the constraints due to a variable change. The gradients are found by repetitively reanalyzing the layout. The allowable value of the stress constraints will change with the design variables. The allowable stresses are updated in accordance with the US Army Corps of Engineers document: BASIC PILE GROUP BEHAVIOR (10). See Appendix 1 for further details on the analysis of pile groups.

3.1 PILE ORIENTATION PROBLEM

The layout of piles under dam structures, retaining walls, and control towers is to be optimized. The cross-sections and the orientations of the piles will be varied. The initial coordinates of the head of the piles will be fixed. The batter is the orientation that will vary. The batter of the piles is the ratio of the depth of the pile to the horizontal distance between the pile head and tip as shown in Figure 3-1.

Variables in the optimization process can be varied either discretely or continuously. In a continuous process the variables may acquire any value within the design space.

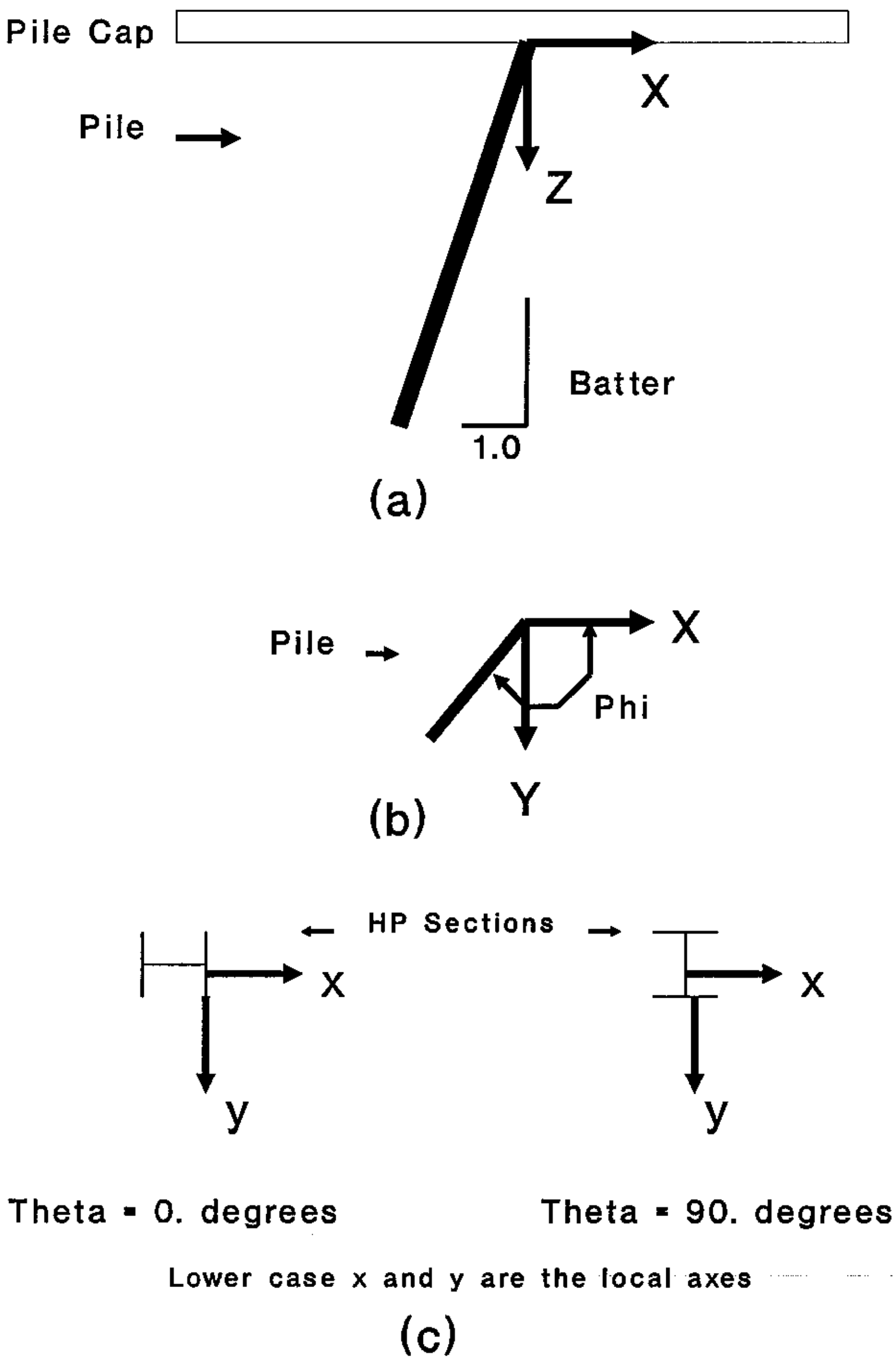


Figure 3-1. Pile coordinate system.
(a): Batter, (b): Phi, (c): Theta.

In a discrete process the variables are allowed to acquire only values contained within a specified set of values.

The cross-sections will not be restricted to discrete sizes but will be governed by a continuous set of equations which describe the HP-14 sections. It is required that one variable should entirely represent the cross-section in the optimization process. All the cross-sectional properties will be expressed in terms of one primary variable which is the major axis moment of inertia (I_{xx}). The secondary variables are the minor axis moment of inertia (I_{yy}), the area (A), and the extreme fiber distances (c_x, c_y). The following equations are approximations developed from the AISC Manual for HP-14 sections (11). See Figures 3-2 to 3-5.

$$I_{yy}=0.37067 I_{xx} -9.2200 \ (inches^4) \tag{3.1}$$

$$A=0.049153 I_{xx}^{0.92180} \ (inches^2) \tag{3.2}$$

$$c_x = 1.2110*10^{-3} I_{xx} + 12.7192 \ (inches) \tag{3.3}$$

$$c_y = 6.110*10^{-4} I_{xx} + 14.139 \ (inches) \tag{3.4}$$

The pile orientation problem is subject to several types of constraints such as maximum member stress limits, displacement limits, soil capacity, maximum and minimum member sizes, and the layout restrictions. The first layout restriction demands that the tips of two piles may not come into contact with each other. In two-dimensions the tip

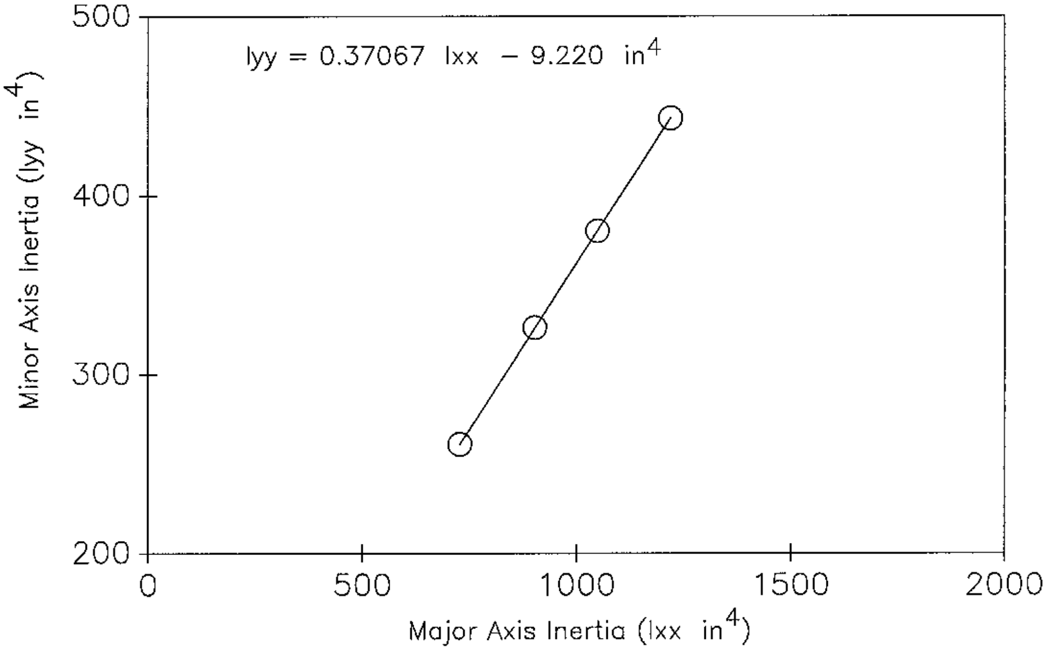


Figure 3–2. Minor Axis Inertia vs. Major Axis Inertia

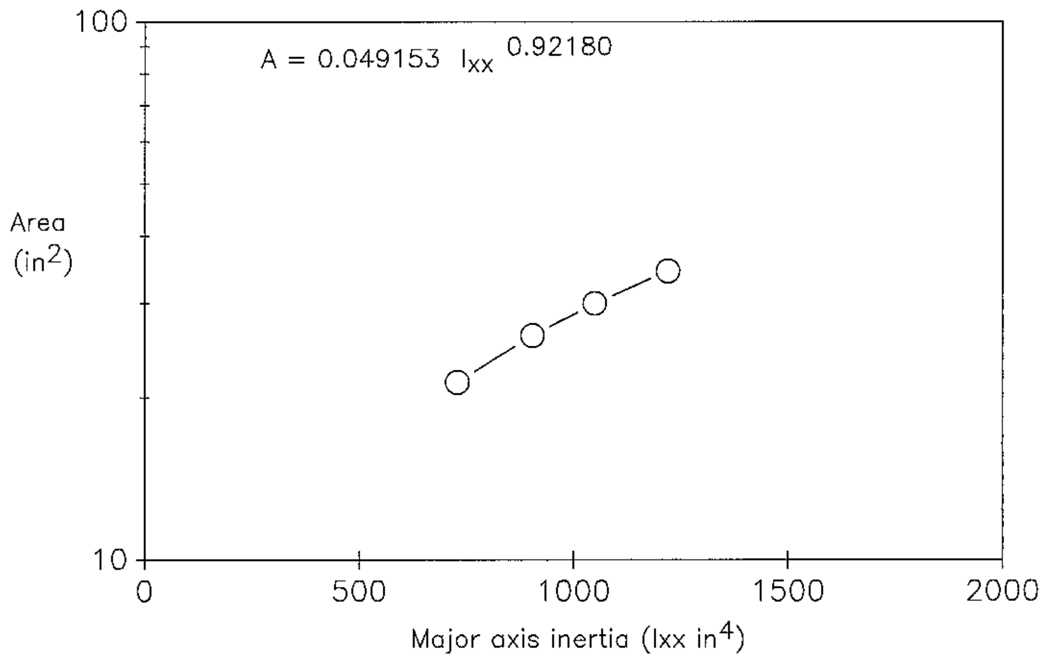


Figure 3–3. Area vs. Major Axis Inertia

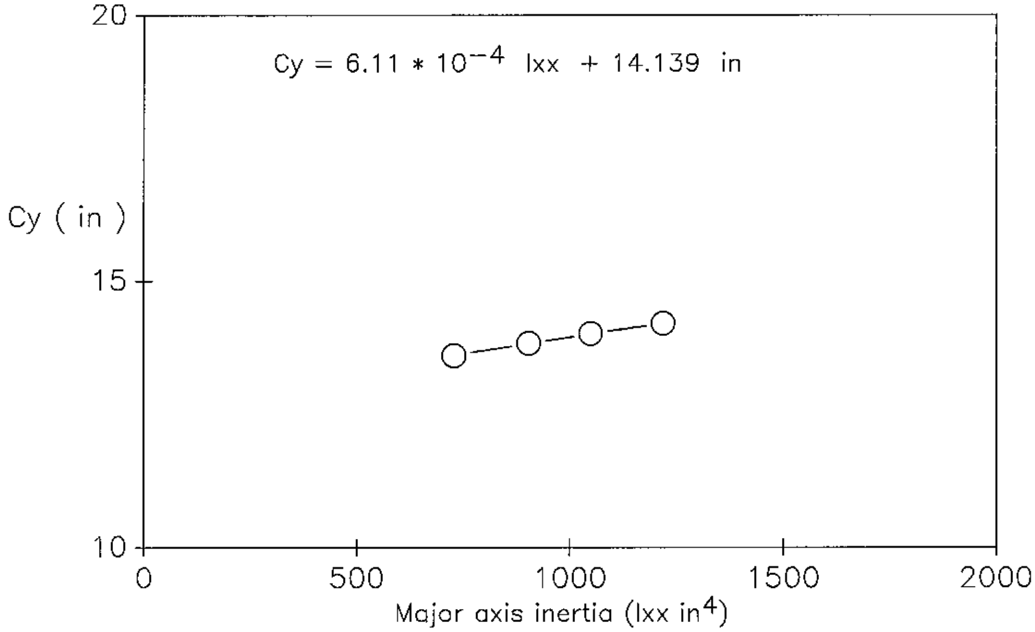


Figure 3–4. Extreme fiber distance in y–direction vs. major axis inertia

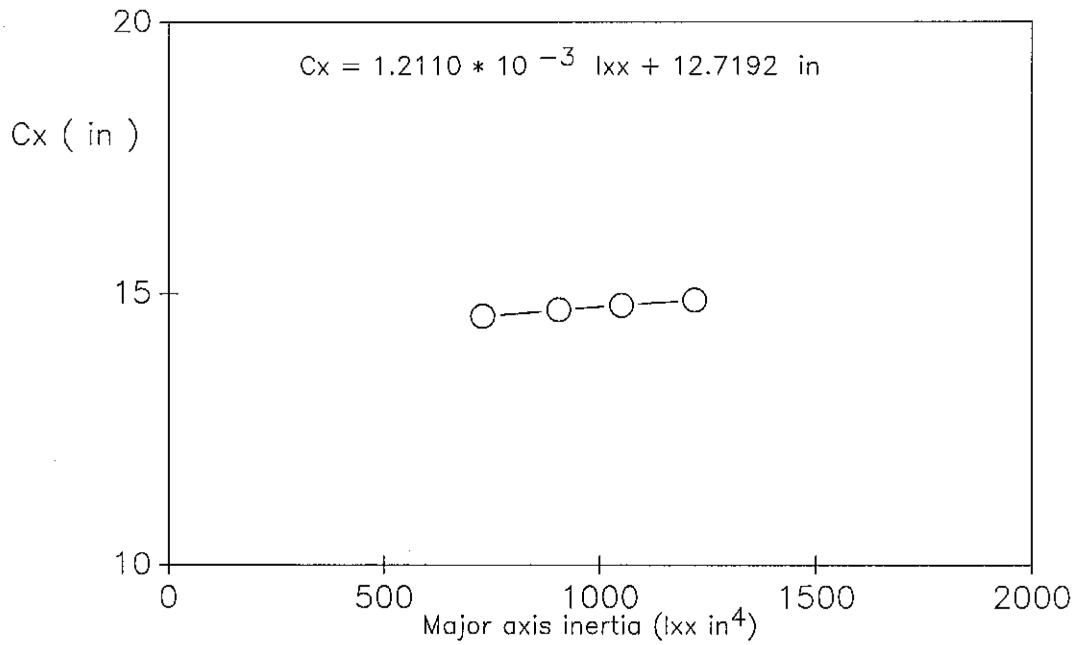


Figure 3–5. Extreme fiber distance in x–direction vs. Major axis inertia

constraint is applied by constraining the coordinate of one tip to be always greater than the coordinate of the second tip. Another restriction is that piles may not be battered at angles that are far from the vertical position. This is called a minimum batter constraint.

3.2 APPLICATION OF OPTIMALITY CRITERIA

The optimization algorithm needs the gradients of the constraints and the objective function. The gradients are approximated using a finite difference method. Each of the design variables is altered to find the resulting change in the constraints.

3.2.1 ENERGY OBJECTIVE FUNCTION

The stored energy resulting from global forces and displacements will be used as an alternate objective function for cases when the weight objective is inadequate. It is stated as:

$$W_T = \sum_{i=1}^{6} P_i D_i \qquad (3.5)$$

where P_i is the global force, D_i is the global displacement for component i, and W_T is the new artificial objective function.

Piles that are vertical have a weight gradient which is zero for a change in batter. A non-zero weight gradient is needed by the algorithm to form the linear Lagrange Multiplier equations. An arbitrary objective which is small relative to the other gradient components could be used. The small weight gradient would reflect that the variable has no direct effect on the weight. This arbitrary gradient should be large enough so that the solution is not numerically ill-conditioned. When energy is taken as an alternate objective of the vertical piles then the energy gradients can be scaled to a level so that they are slightly significant. The energy gradients of the batters are scaled to one percent of the value of the weight gradient for a batter that is not vertical.

Using the energy as an objective function for the vertical piles provides a greater meaning than using a completely arbitrary objective function. An energy objective function is effectively a displacement objective function since the global forces are constant and only displacements will vary. One reason the energy objective has meaning is because small global cap displacements are desirable. Another reason that this objective is proper is that piles have less displacement when they are nearly aligned with the load. The resulting alignment would help the pile to resist the loads more efficiently by axial deformation rather than by flexure.

In problems where piles are allowed to rotate in all three dimensions the variable of rotation about the vertical axis would not have a weight change associated with it. An alternate objective such as an energy objective could be used for these variables. This variable is not considered in the examples.

3.2.2 TOPOLOGICAL VARIABLES

The variables which represent the batter of the piles may be measured by using either the angles of rotation or the coordinates of the tip of the piles. Using the tip coordinates as the variables would simplify the pile tip interference constraints.

Each batter is given it's own local coordinate system. In this report the distance between the local coordinate system and the current tip position will be referred to as slack.

The choice of the origin of the coordinate system will have an effect on the convergence of each variable. Consider this example: A variable which has nothing restricting it's change will prefer to passively decrease itself to zero. If the convergence control parameter (r) is equal to two then the variable would halve itself. If this variable is a topological variable with a slack of 80 feet then the tip would move 40 feet, however the pile behavior may be predictable for a change of only 30 feet. The slack should be chosen given the nature of the variable and it's gradients. If the constraints vary linearly with the variables then the variables have well behaved gradients. This greater predictability would allow an increased slack.

Certain topological variables improve the objective function as their value increases. They have a negative weight gradient. An unrestricted variable with a negative weight gradient should increase. Given the unaltered algorithm all variables which are not actively controlled by a constraint would decrease towards the origin of the local coordinate system. When r is two they would halve themselves.

The variable recurrence formula appears to need reformulation. It was developed using the optimality criteria as a measure of the efficiency. See Equations 2.12 and 2.13. When the weight gradient is larger than the restraining components the variable is unrestricted; which is true for any weight gradient value. The negative coefficients of the negative weight gradient cause the resulting optimality criteria value to be greater than 1 instead of less than 1.

Two methods could be used to adapt the optimization method without altering the recurrence equations. The first is to simply reverse the sense of the local coordinate system so that the weight gradients are always positive. The method which is used consists of applying negative slack while maintaining the sense of the coordinates. This method is chosen so that all of the gradients do not have to be switched in sign and a small amount of computer time is saved. For this method when a variable is not controlled and when r is two it would halve itself. The negative variable would increase towards the origin.

4. NUMERICAL RESULTS

The examples optimized the pile layouts for: one retaining wall, four dams, and one control tower.

Each pile does not have an independent cross-section and orientation. The piles are linked into groups which share the same cross-section and batter. This will ease installation of the piles, and it will reduce the size of the optimization problem to be solved.

TYPES OF CONSTRAINTS

1. Member stress constraints. The most highly stressed member in any group is prevented from becoming over-stressed.

2. Size constraints. The cross-sections are prevented from advancing beyond the maximum or minimum section sizes available.

3. Interference constraints. Piles can not pass through one another. In two-dimensional problems and the less complicated three-dimensional problems this constraint can be established by preventing the pile tips from meeting.

4. Minimum batter constraints. The piles may not be battered parallel to the ground surface or beyond a specified angle.

5. Displacement constraints. Limits on displacements are thought to aide problem convergence by preventing bizarre estimates of the optimum point.

In these examples the most frequently active constraints at the optimum are the member stress, and member size constraints.

4.1 EXAMPLE 1.

Example 1. This example is a retaining wall. The example's symmetry of geometry and loading allows a segment of the wall to be modeled as a two-dimensional problem. The segment has five piles arranged into two groups as shown in Tables 4-1,2 and Figure 4-1.

The final layout is found in Table 4-3, and Figure 4-2. This example reaches a convergent solution in five iterations of the optimality criteria algorithm. The weight improves from 15294. to 12766. lbs as shown in Figure 4-3. In the first iteration the weight increases because the original layout has a 44% stress constraint violation in the second load case.

The active constraints at the optimum are the minimum inertias of both groups, and the stresses in members 3 and 1 due to load cases 1 and 2 respectively. See Figure 4-4 for the convergence of the stress constraint for member 1.

Table 4-1. Loading for examples 1.,1.4, and 1.5.

Load case	P_x kips	P_y kips	P_z kips	M_x ft-k	M_y ft-k	M_z ft-k
1	-120.	0.	340.	0.	500.	0.
2	150.	0.	180.	0.	0.	0.

Table 4-2. Initial layouts and sizes for examples 1., 1.1, 1.2, and 1.3.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	1.	180.	0.
2	729.	1.	0.	0.

Table 4-3. Final layouts and sizes for example 1.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	4.69	180.	0.
2	729.	1.04	0.	0.

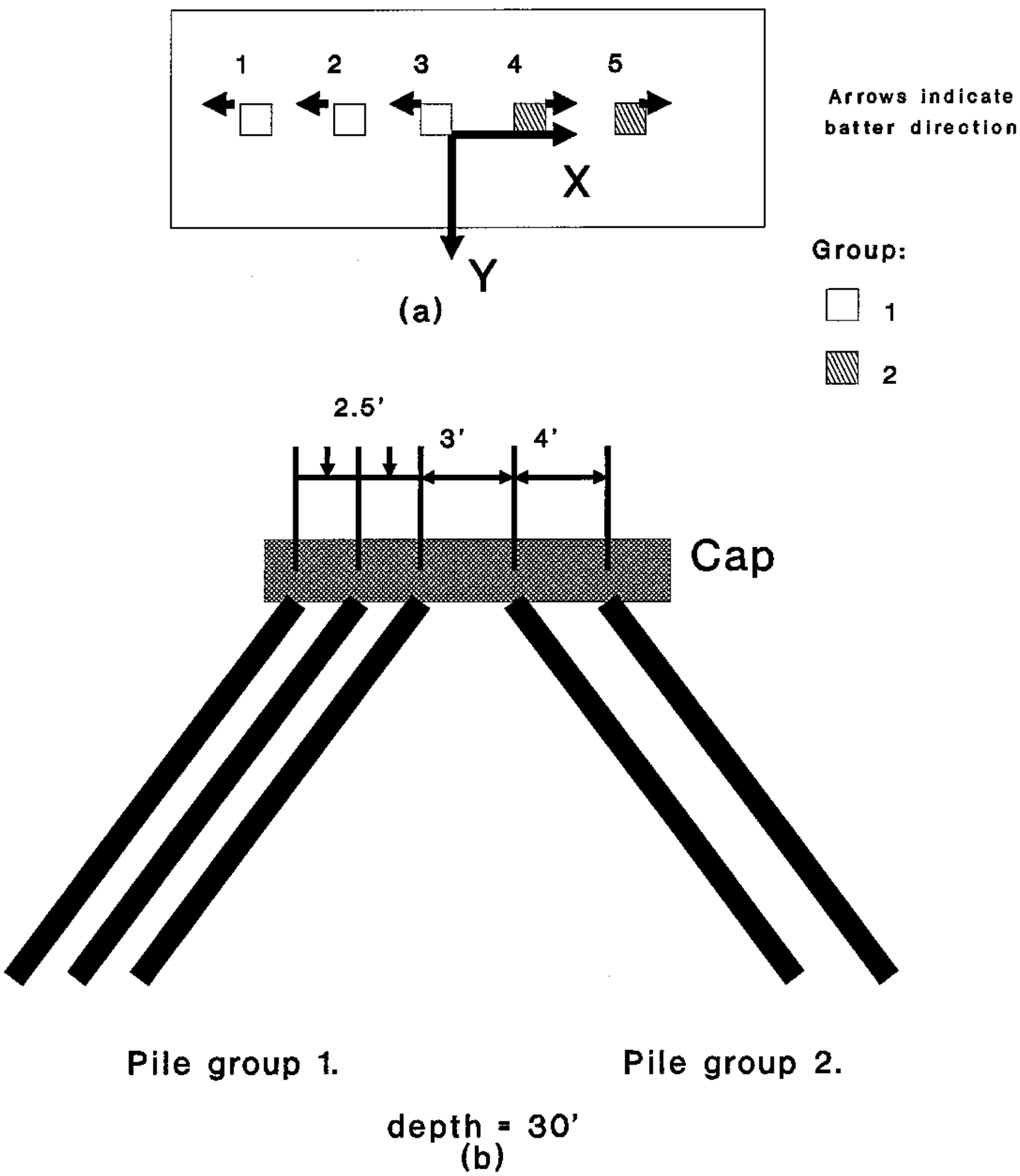


Figure 4-1. Initial pile layouts for examples 1.,1.1,1.2, and 1.3. (a):Plan view, (b):Elevation.

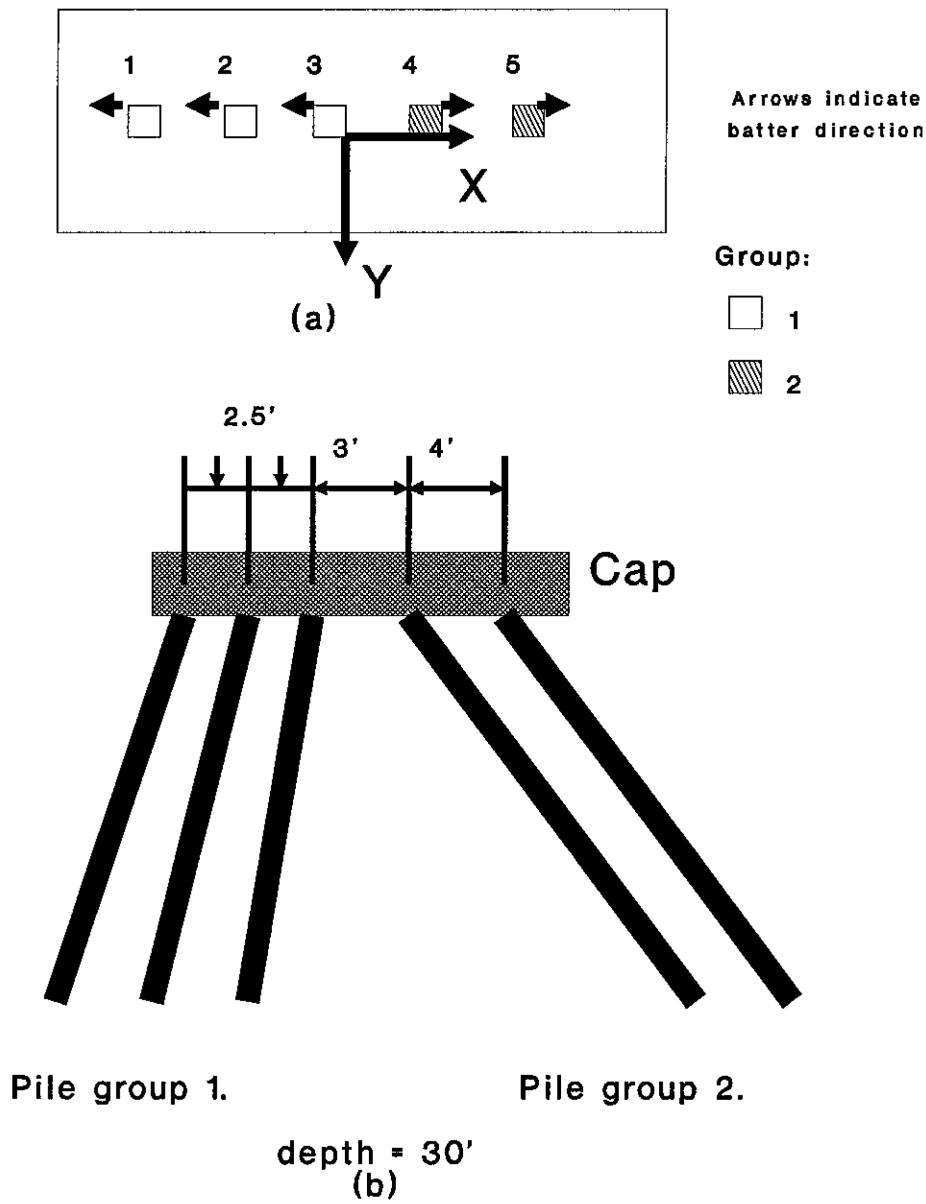


Figure 4-2. Final pile layouts for example 1. (a):Plan view, (b):Elevation.

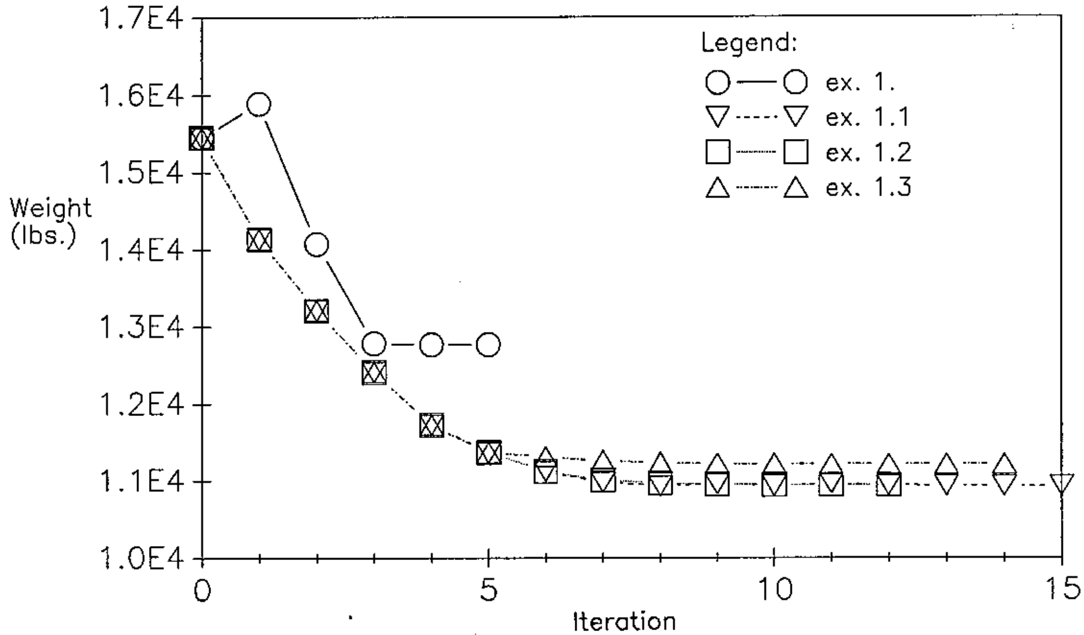


Figure 4—3. Convergence of weight for examples 1.—1.3.

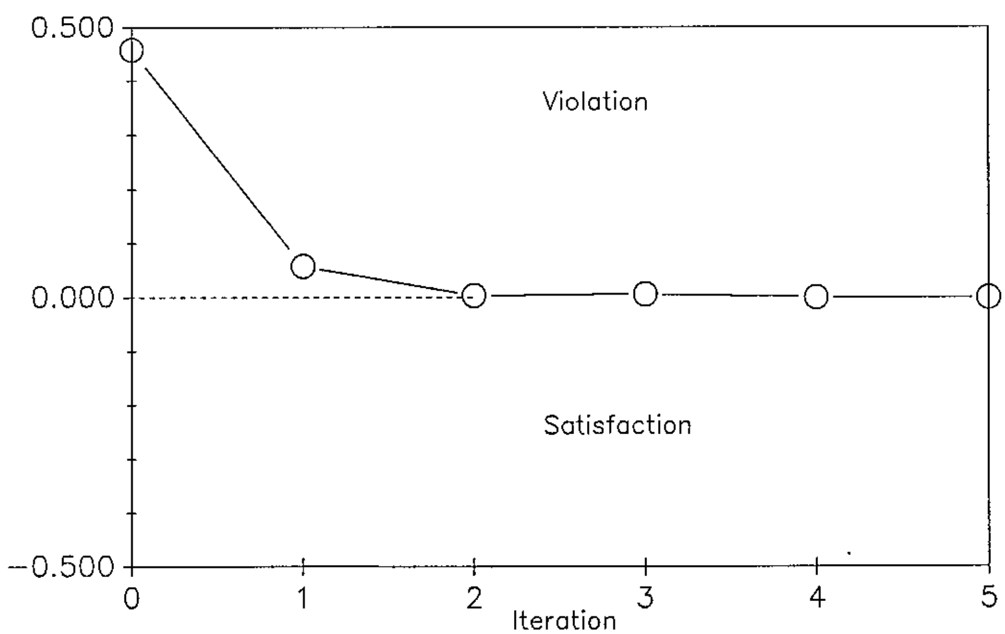


Figure 4—4. Example 1.1, Convergence of stress constraint for pile number 1, load case 2.

Example 1.1: This example has the geometry of example 1, but the loading is much less. The loading was arbitrarily reduced as shown in Table 4-4. Various versions of example 1 were optimized to explore the behavior of the optimization process. The loading for this example was reduced to be able to view the optimization performance for a design which is originally too conservatively designed. An optimization algorithm should significantly reduce the weight for a initially conservative design.

The final layout is found in Table 4-5, and Figure 4-5. The weight improves from 15294. to 10924. lbs after 15 iterations as shown in Figure 4-3. The minimum size for the two groups are the only constraints that are active. The loading is so small that the pile batters become vertical to minimize the weight. The pile batters are not required to be outwardly positioned to be aligned with the load vector.

Table 4-4. Loading for examples 1.1 - 1.3.

Load case	P_x kips	P_y kips	P_z kips	M_x ft-k	M_y ft-k	M_z ft-k
1	-39.375	0.	113.1	0.	173.4	0.
2	50.	0.	60.	0.	0.	0.

Table 4-5. Final layouts and sizes for example 1.1.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	100.	180.	0.
2	729.	100.	0.	0.

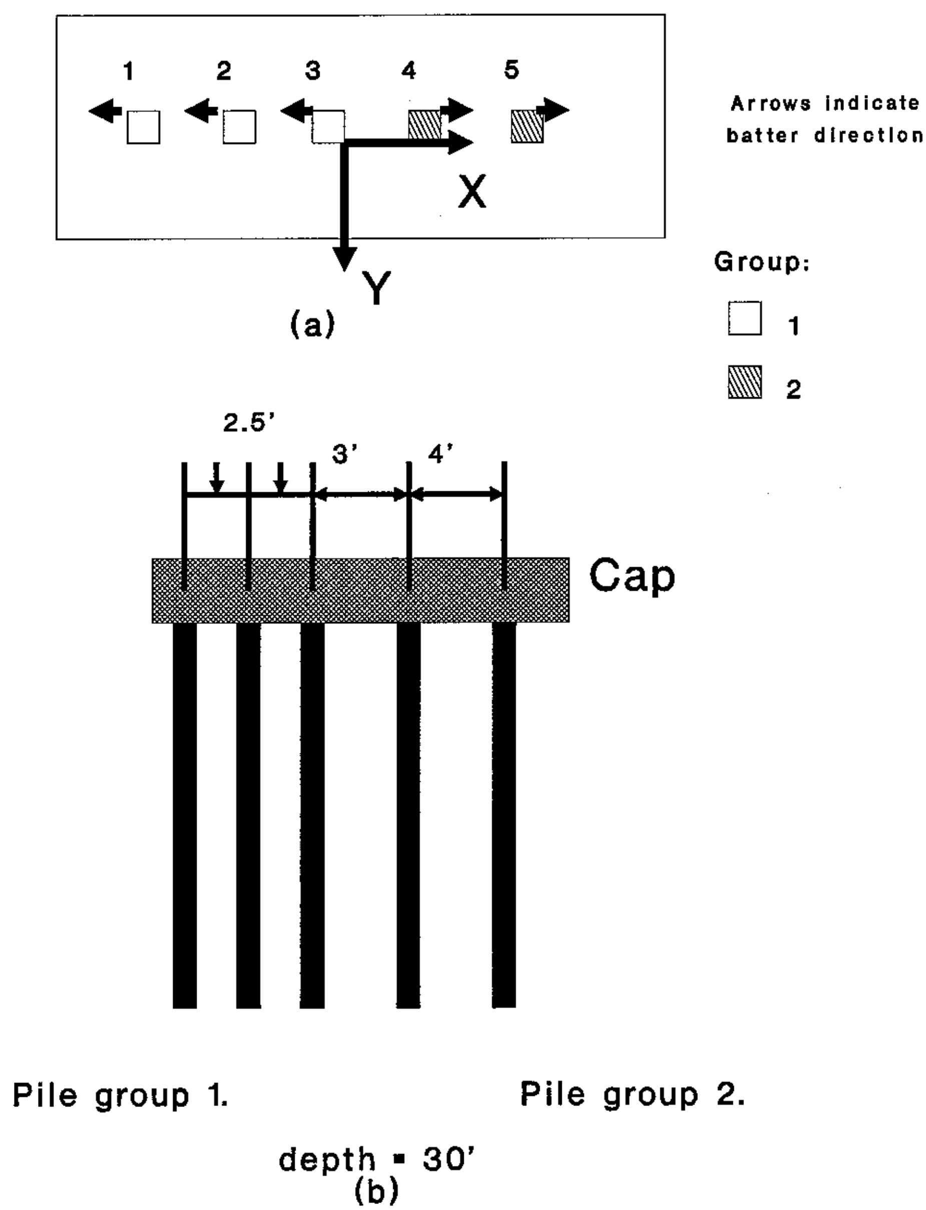


Figure 4-5. Final pile layouts for example 1.1
(a):Plan view, (b):Elevation.

Example 1.2: This example is example 1.1 with a pile tip interference limit of six feet. Interference constraints are not frequently active in the following examples. The minimum distance of six feet between the pile groups was expected to force this constraint to become active.

The final layout is found in Table 4-6, and Figure 4-6. After 12 iterations a final weight of 10939. lbs is reached as shown in Figure 4-3. Note the weight is very close to the previous example. It is expected that making constraints more strict would increase the weight. The interference constraints simply force the batters from the vertical positions. The insignificant weight gain demonstrates insensitivity of the weight to the batters.

The active constraints are the same as example 1.1 with the addition of the interference constraint. The convergence of the interference constraint is found in Figure 4-7.

Table 4-6. Final layouts and sizes for example 1.2.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	20.3	180.	0.
2	729.	18.3	0.	0.

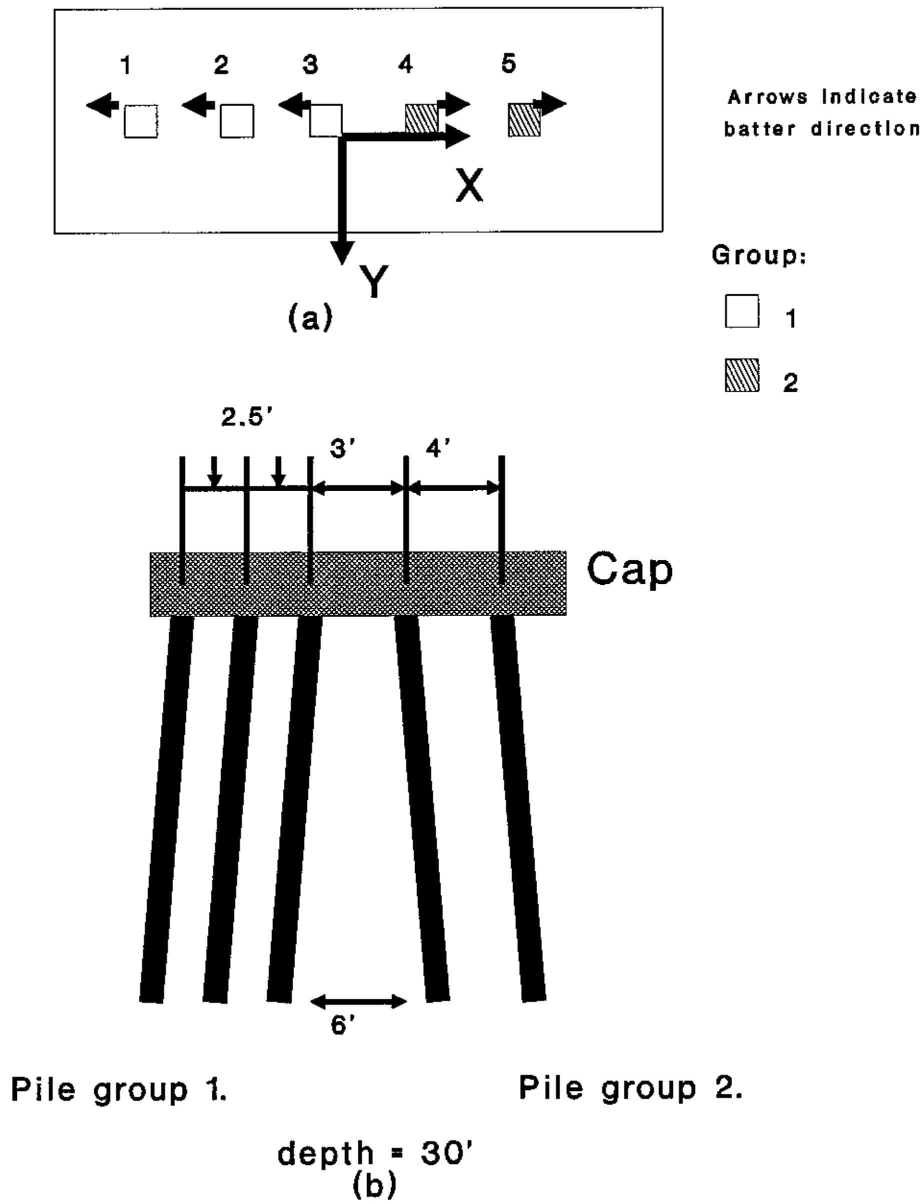


Figure 4-6. Final pile layouts for example 1.2
(a):Plan view, (b):Elevation.

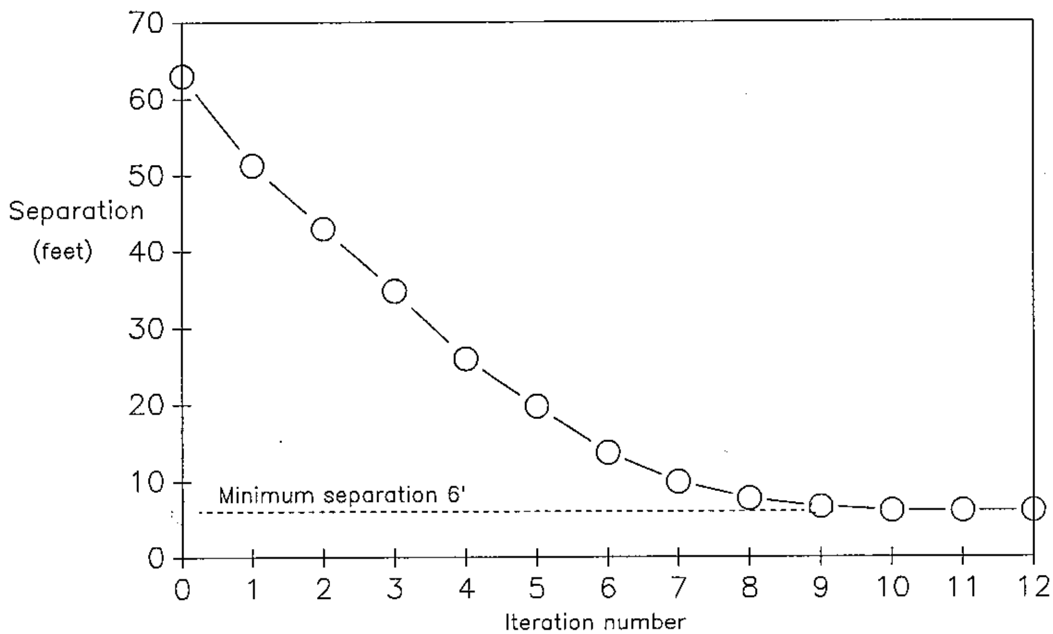


Figure 4—7. Example 1.2, Separation between piles 3 and 4.

Example 1.3: In this example the displacements are restricted to view this type of constraint when active. The displacement in the x-direction is restricted to 0.10 inches. See the initial layout in Tables 4-2,4 and Figure 4-1.

The final layout is found in Table 4-7, and Figure 4-8. After 14 iterations the weight converges to 11211. lbs. The weight increased slightly compared to example 1.1 as shown in Figure 4-3. The displacement constraint in addition to the constraints of example 1.1 are active. The displacement is precisely controlled. The displacement convergence is found in Figure 4-9.

Table 4-7. Final layouts and sizes for example 1.3.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	13.7	180.	0.
2	729.	2.81	0.	0.

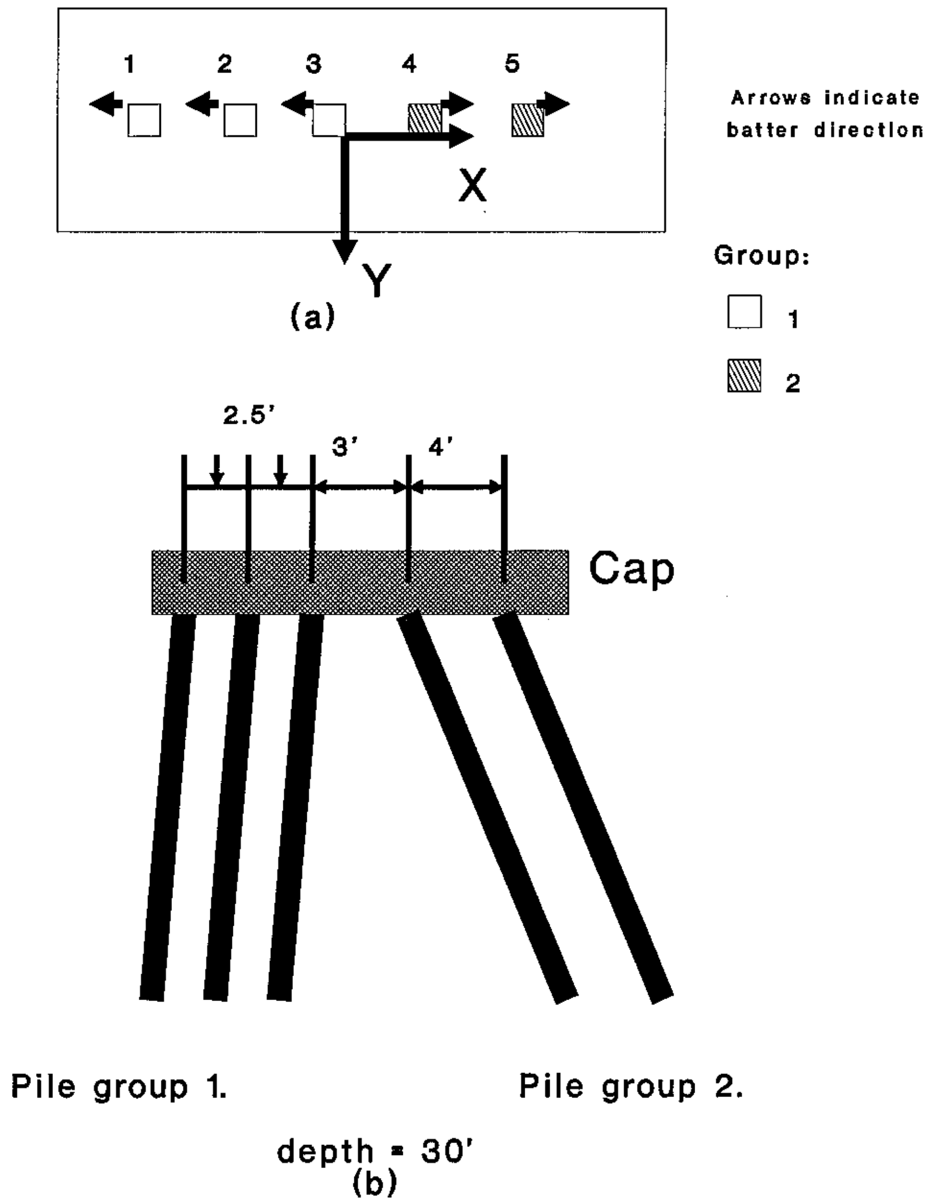


Figure 4-8. Final pile layouts for example 1.3
(a):Plan view, (b):Elevation.

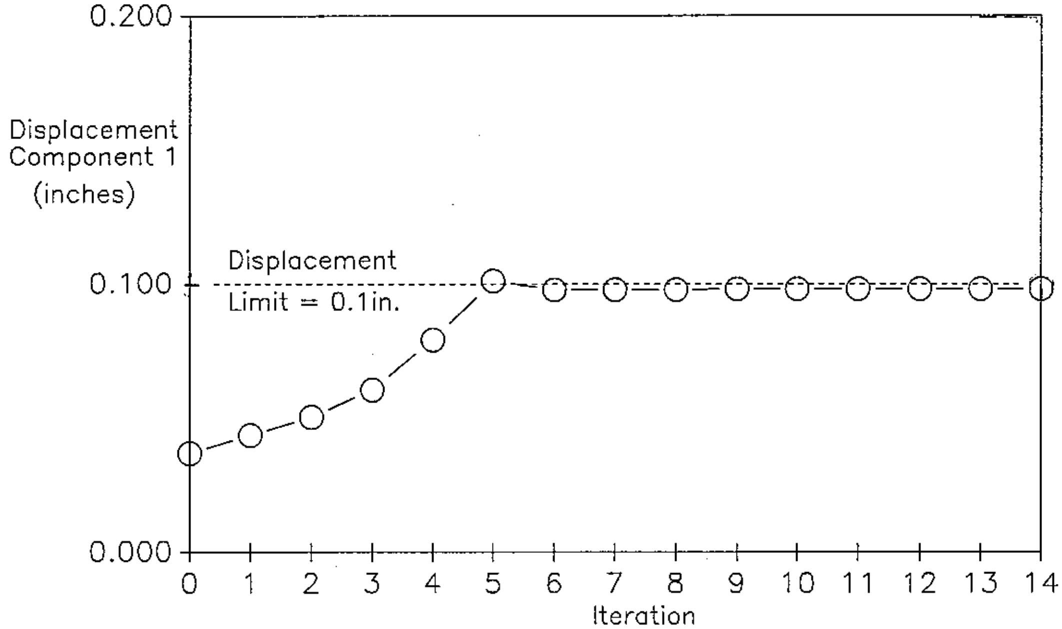


Figure 4–9. Satisfaction of displacement constraint for example 1.3.

<u>Example 1.4:</u> The choice of linking piles into groups is performed by the designer. It should be determined if the best grouping has been chosen. In this version of example 1 the piles are unlinked. Their final layout should give insight into the best grouping. The pile tips are constrained to be separated by at least two feet.

See the initial and final designs in Tables 4-1,8,9 and Figures 4-10,11. After 11 iterations the weight has converged from 12799. to 11753. lbs as shown in Figure 4-12. As expected, this is slightly smaller than the 12766. lbs. for example 1. with linked piles. The active constraints are: the minimum size for all groups; stress in piles 1 and 2 for load cases 1 and 2 respectively; and the interferences of piles 2 through 5.

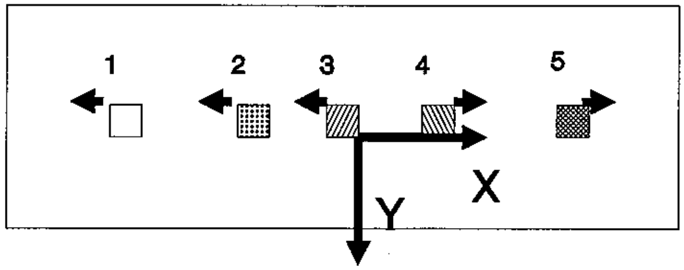
The final pile layout is not the same as in the linked examples. Pile (1) is orientated outward. The other piles (2,3,4,5) are more vertical than pile (1). This relatively small loading does not require all piles to be orientated outward. For this magnitude of loading a better grouping might be the central piles in one group and Pile (1) in second group.

Table 4-8. Initial layouts and sizes for examples 1.4, and 1.5.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	1.	180.	0.
2	729.	5.	180.	0.
3	729.	10.	180.	0.
4	729.	10.	0.	0.
5	729.	1.	0.	0.

Table 4-9. Final layouts and sizes for example 1.4.

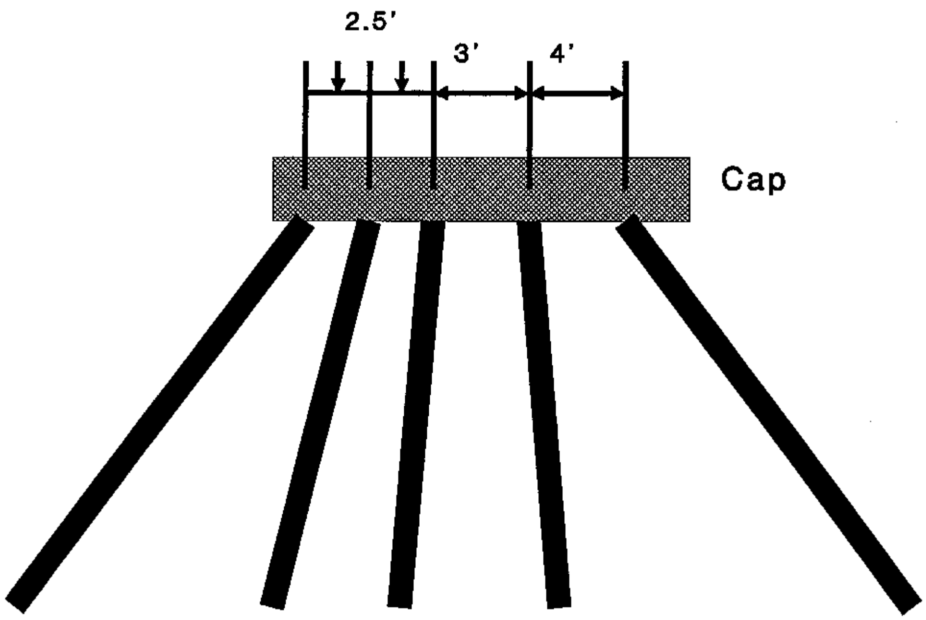
Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	1.30	180.	0.
2	729.	3.51	0.	0.
3	729.	3.71	0.	0.
4	729.	4.21	0.	0.
5	729.	5.80	0.	0.



{a}

Arrows indicate batter direction.

Piles are ungrouped.



depth = 30'

{b}

Figure 4-10. Initial pile layouts for examples 1.4 and 1.5.
{a}:Plan view, {b}:Elevation.

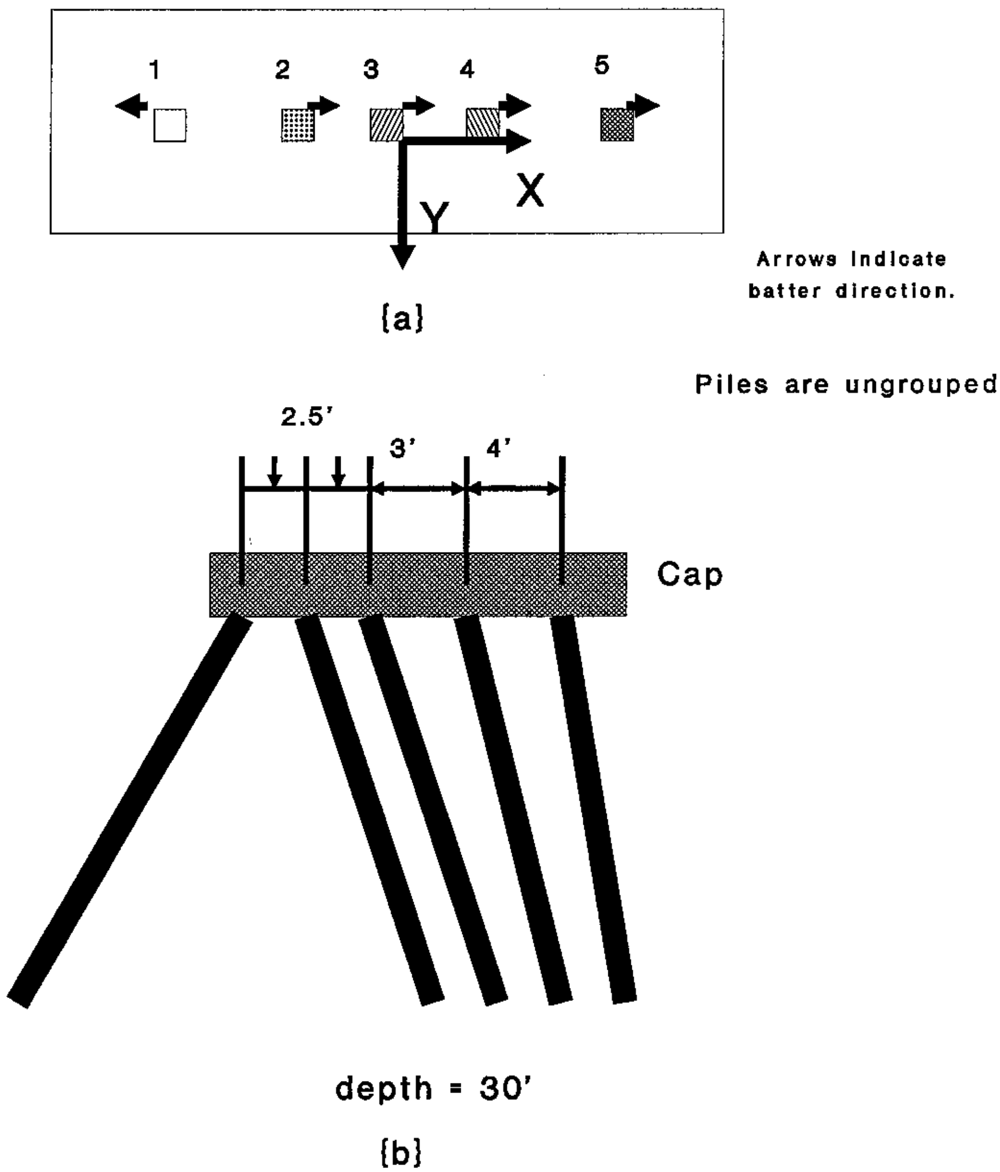


Figure 4-11. Final pile layouts for example 1.4
{a}:Plan view, {b}:Elevation.

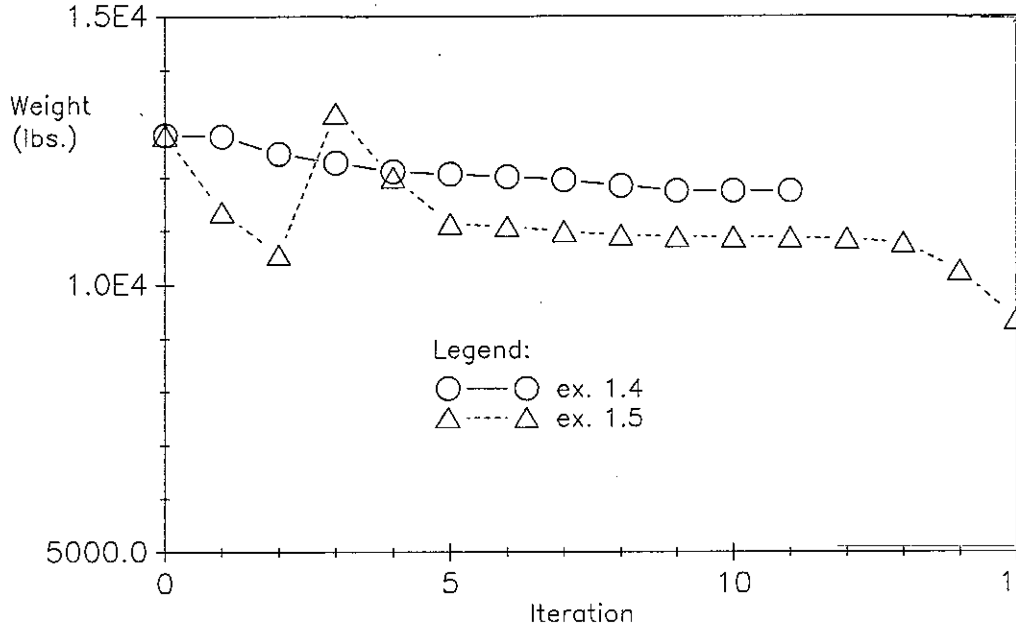


Figure 4-12. Convergence of weight for examples 1.4 and 1.5.

Example 1.5: Some of the best weight reductions are expected from the elimination of piles from the design. The elimination of a pile can be done by discrete methods. A continuous method for the elimination of piles was attempted. It was hoped that elimination of the minimum member size constraints would allow a member to reduce itself in size until it reaches zero. In this example there is no minimum member size constraints. See the initial layouts in Tables 4-1,8 and Figure 4-10.

See the final layout in Table 4-10, and Figure 4-13. After 15 iterations the weight reduces to 9360. lbs from 12799. lbs as shown in Figure 4-12. The weight increased in the third iteration because the first three layouts had stress constraint violations.

The members have reduced in size to a roughly equivalent value. It is not obvious if a member can be eliminated. This method of member elimination is not satisfactory for this example. It is expected that for larger examples that this method will behave more beneficially.

From Figure 4-12 it is possible to achieve further weight reductions by continuing the optimization process. As stated above the method of this example will not be used therefore continuation of the process is unnecessary.

The active constraints of the final layout are: the tip interferences for piles 2 through 5; the minimum batter for pile 1; and the stresses in piles 1,5,2, and 3 for load cases 1,1,2, and 2 respectively. Note that many stress constraints are active. Since the member sizes are not limited the optimization algorithm improves the weight until other constraints become active.

Table 4-10. Final layouts and sizes for example 1.5.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	800.8	1.00	180.	0.
2	543.3	2.34	0.	0.
3	451.2	2.42	0.	0.
4	449.9	2.63	0.	0.
5	349.4	3.18	0.	0.

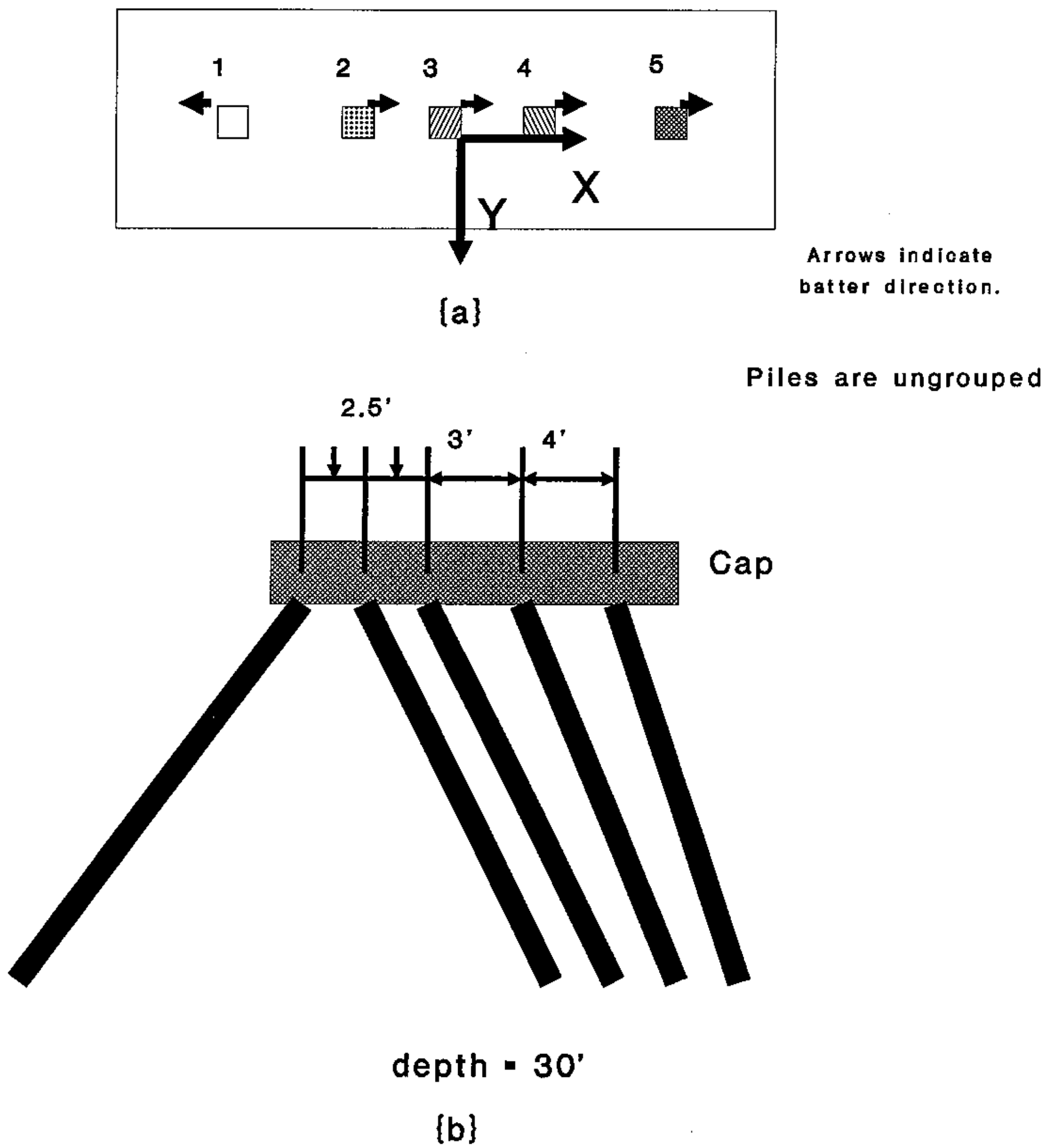


Figure 4-13. Final pile layouts for example 1.5
{a}:Plan view, {b}:Elevation.

4.2 EXAMPLE 2.

Example 2. This example has eight piles. None of the piles are linked to any of the other piles. The piles are arranged in an unsymmetrical pattern as shown in Tables 4-11,12 and Figure 4-14. The minimum size of the cross-sections was not constrained. This example was a second attempt to eliminate piles by allowing them to reduce to zero. This example has the largest number of groups and therefore the largest number of variables of any of the problems.

The final layout is in Table 4-13 and Figure 4-15. As in example 1.5 the optimization process did not produce a list of piles which could be obviously eliminated. A few of the piles are slightly larger than the others. Piles (2,3,6,8) all have similar moments of inertia and batter directions. Piles (1,4,5,7) also have similar properties. The piles may be assigned to two linked groups.

The weight improved from 53856. to 20341. lbs after 15 iterations as shown in Figure 4-16. While developing the optimization algorithm it was noted that a higher convergence control parameter was required for this example. This is because the cross-section variables have no restrictions on their size. They are free to greatly change in magnitude at each iteration. The higher convergence control parameter restricts the variables from large changes that can result in oscillations. Figure 4-16 shows that the weight had effectively converged after five iterations. After that point in any iteration the weight and the variables do not greatly change. The changes are never small enough to meet the convergence criteria, therefore an even larger convergence control parameter could be used.

The active constraints of the final layout are: the stresses in piles 1,and 7 for load case 1; and all piles for load case 2. The final layout is not a feasible solution. Piles 6 and 7 intersect each other. A tip interference constraint for piles 6 and 7 was not specified as a constraint to consider.

Table 4-11. Loading for example 2.

Load case	P_x kips	P_y kips	P_z kips	M_x ft-k	M_y ft-k	M_z ft-k
1	0.	-240.	320.	-1000.	0.	0.
2	0.	0.	500.	0.	0.	0.

Table 4-12. Initial layouts and sizes for example 2.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1,2,3,4, 5,6,7,8	729.	100.	90.	90.

Table 4-13. Final layouts and sizes for example 2.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	254.0	1.82	270.	90.
2	160.3	1.36	90.	90.
3	166.9	1.39	90.	90.
4	337.8	2.17	270.	90.
5	282.4	2.87	270.	90.
6	148.4	1.15	90.	90.
7	267.4	2.01	270.	90.
8	158.7	1.26	90.	90.

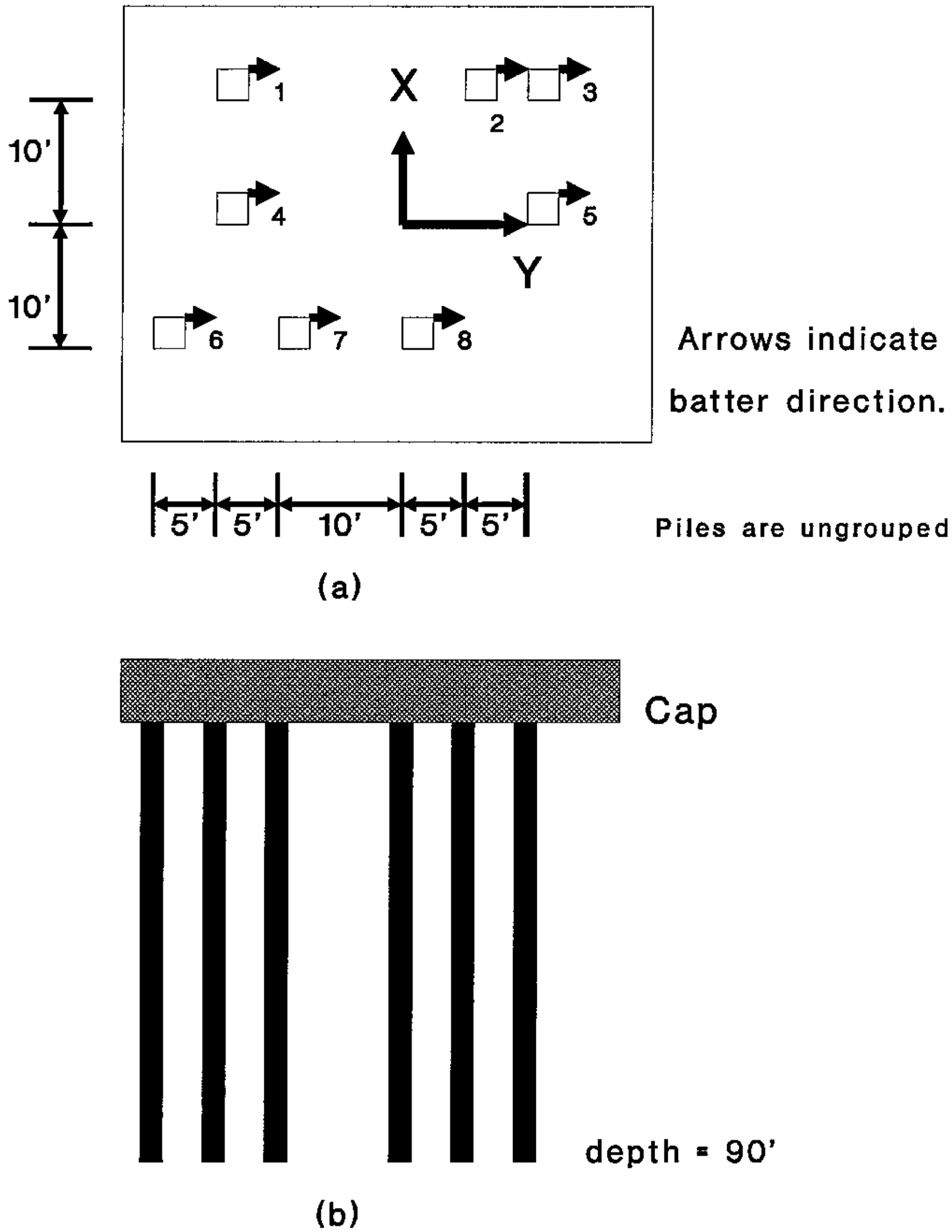
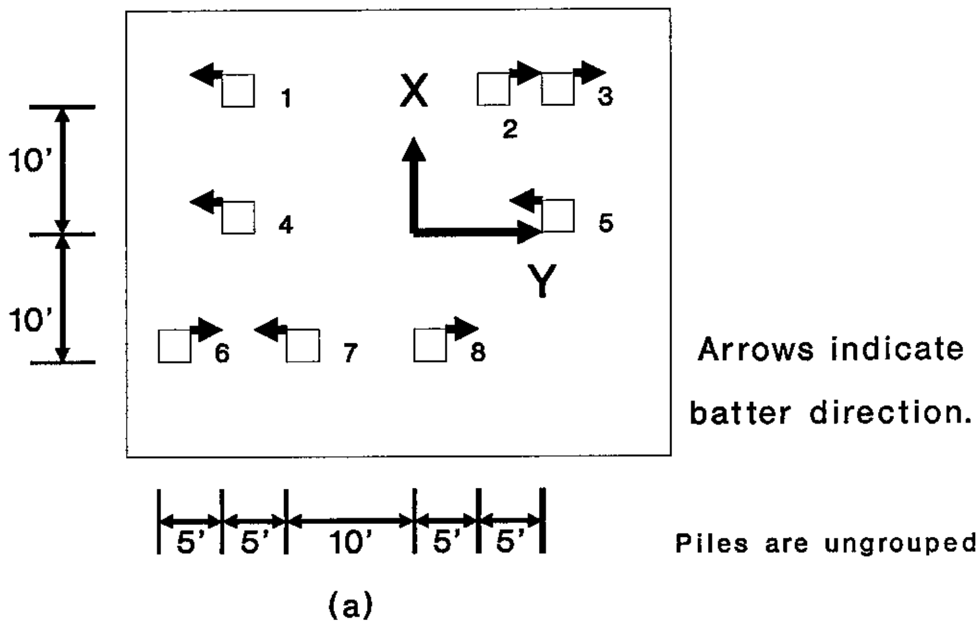


Figure 4-14. Initial pile layouts for example 2.
(a): Plan view, (b): Elevation.



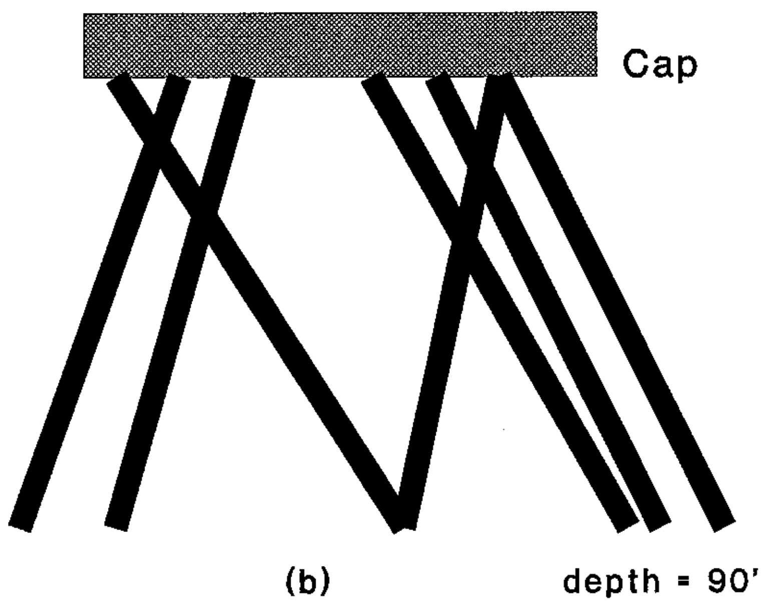


Figure 4-15. Final pile layouts for example 2.
(a): Plan view, (b): Elevation.

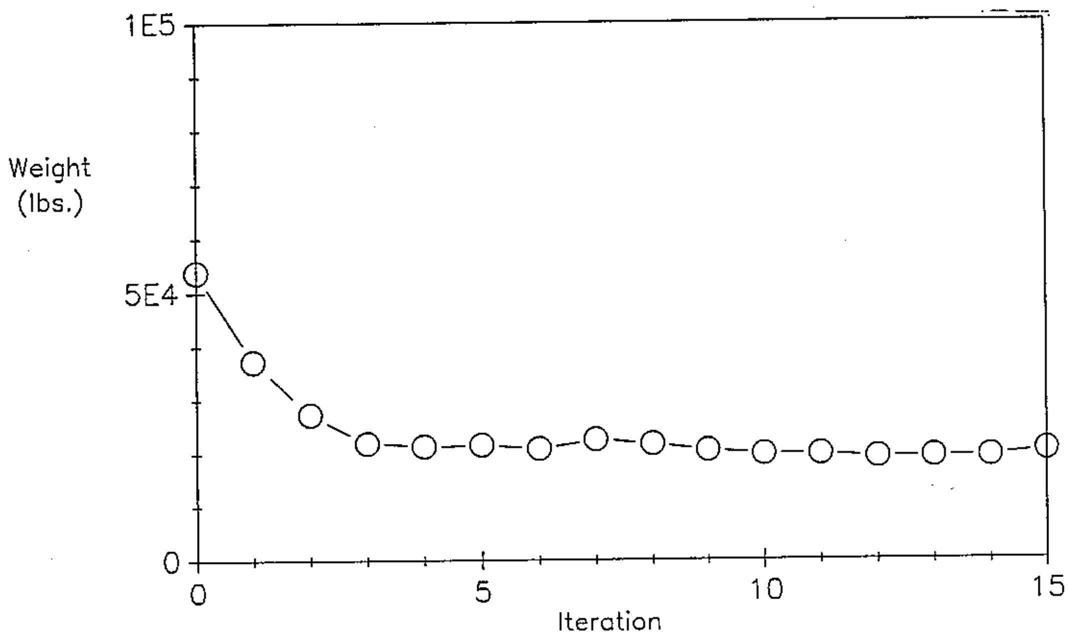


Figure 4–16. Convergence of weight for example 2.

4.3 EXAMPLE 3.

Example 3. This example has 29 piles. The piles are arranged in three rows. A pile is linked to the other piles in the same row as shown in Tables 4-14,15 and Figure 4-17. This is a larger problem and therefore probably a more practical problem than those previously discussed. Since there are only three groups the number of variables is relatively small.

See the final layout in Table 4-16 and Figure 4-18. The weight improved from 237664. to 154912. lbs after 15 iterations as shown in Figure 4-19. The active constraints were the stress in pile 11 for load case 2; and the minimum sizes for all piles. See the convergence of the size of group 3 in Figure 4-20. It was found that in this problem a higher convergence control parameter was required to achieve convergence.

Table 4-14. Loading for examples 3. - 3.2.

Load case	P_x kips	P_y kips	P_z kips	M_x ft-k	M_y ft-k	M_z ft-k
1	0.	-1300.	1500.	-10000.	0.	0.
2	0.	1580.	0.	0.	0.	0.

Table 4-15. Initial layouts and sizes for example 3.0

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	1220.	100.	90.	90.
2	1220.	100.	90.	90.
3	1220.	100.	90.	90.

Table 4-16. Final layouts and sizes for example 3.0

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	3.77	270.	90.
2	729.	2.39	90.	90.
3	729.	4.16	270.	90.

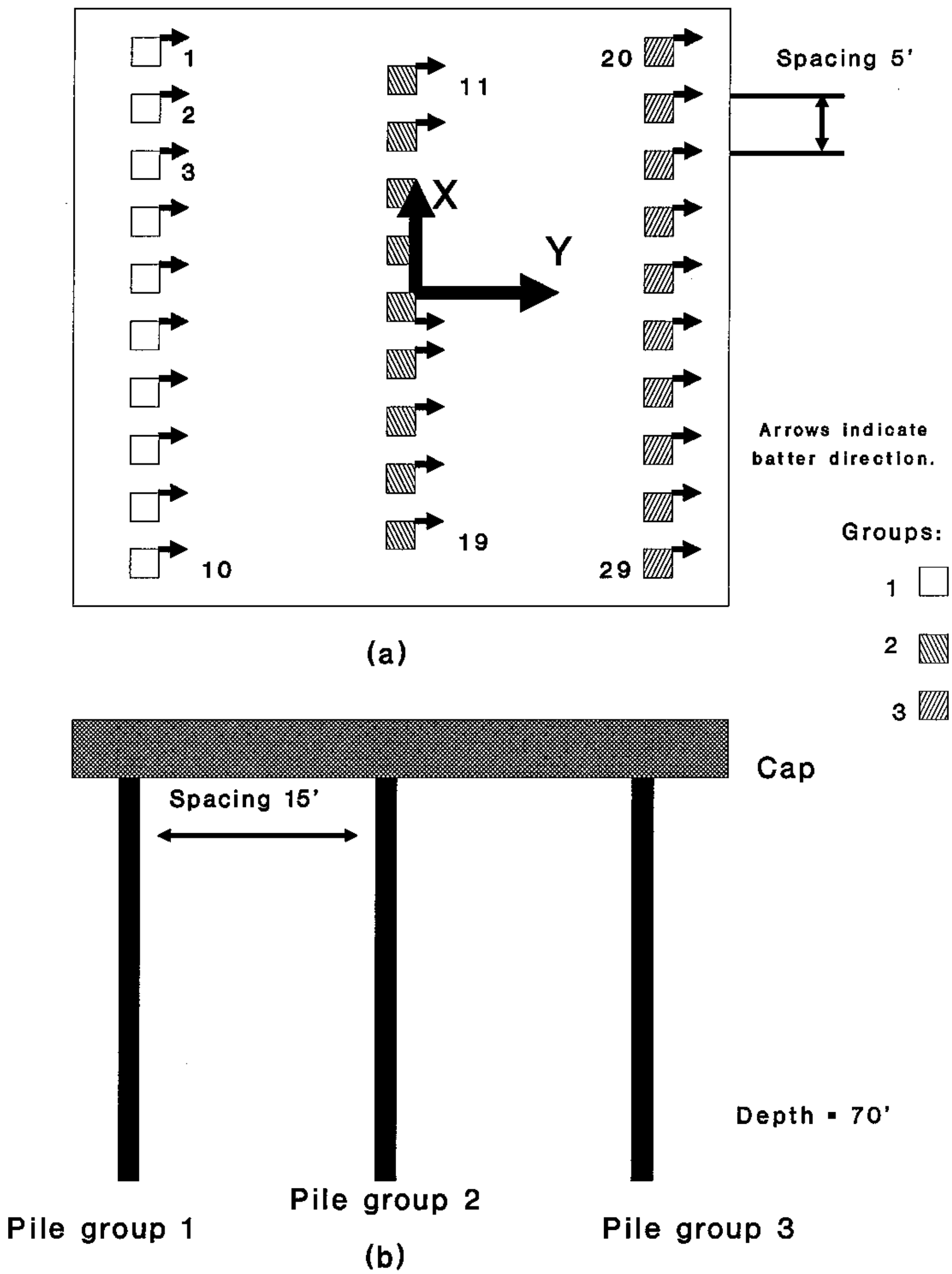


Figure 4-17. Initial pile layouts for examples 3. and 3.1.

(a): Plan view, (b): Elevation.

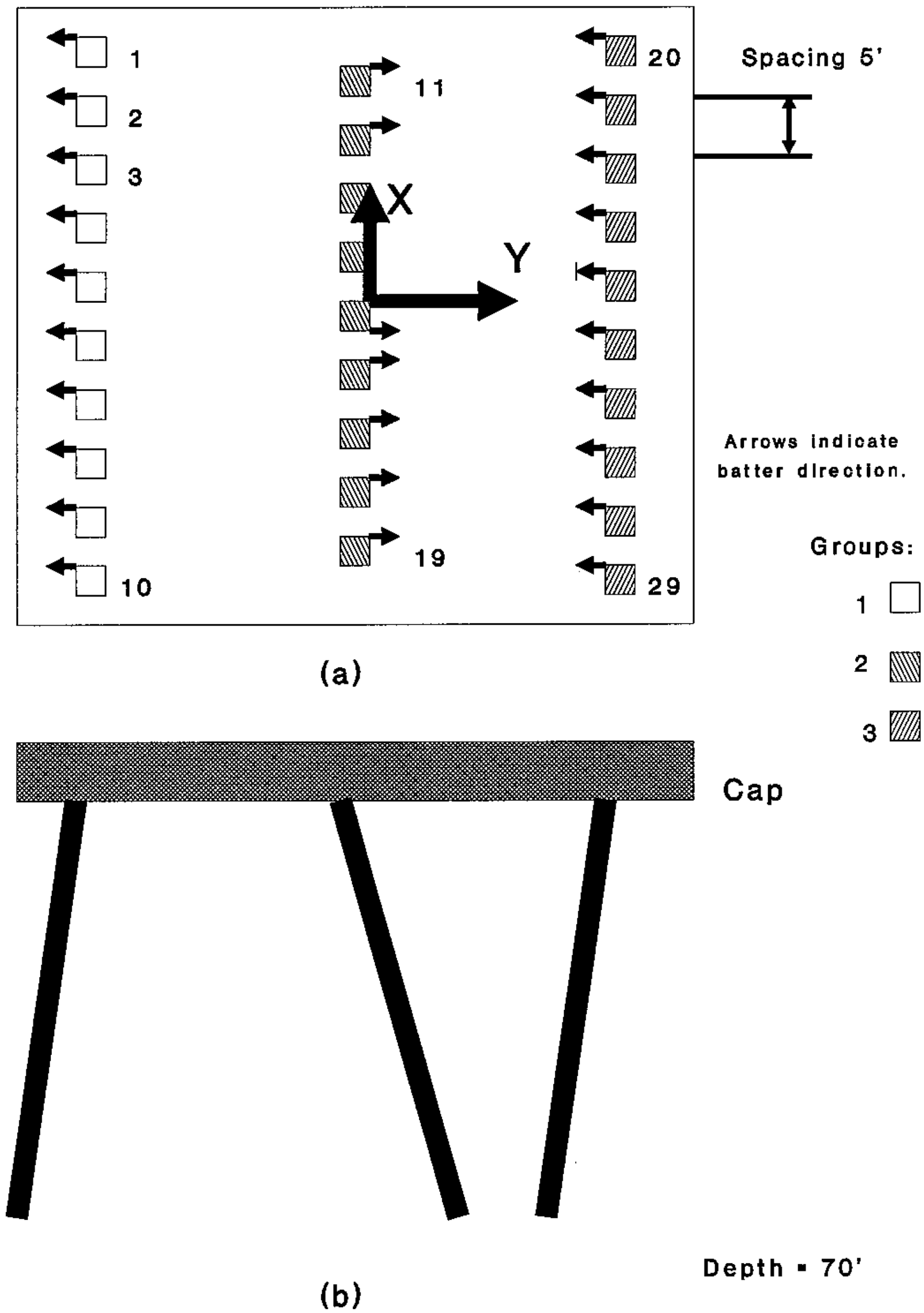


Figure 4-18. Final pile layouts for example 3. and 3.1.

(a): Plan view, (b): Elevation.

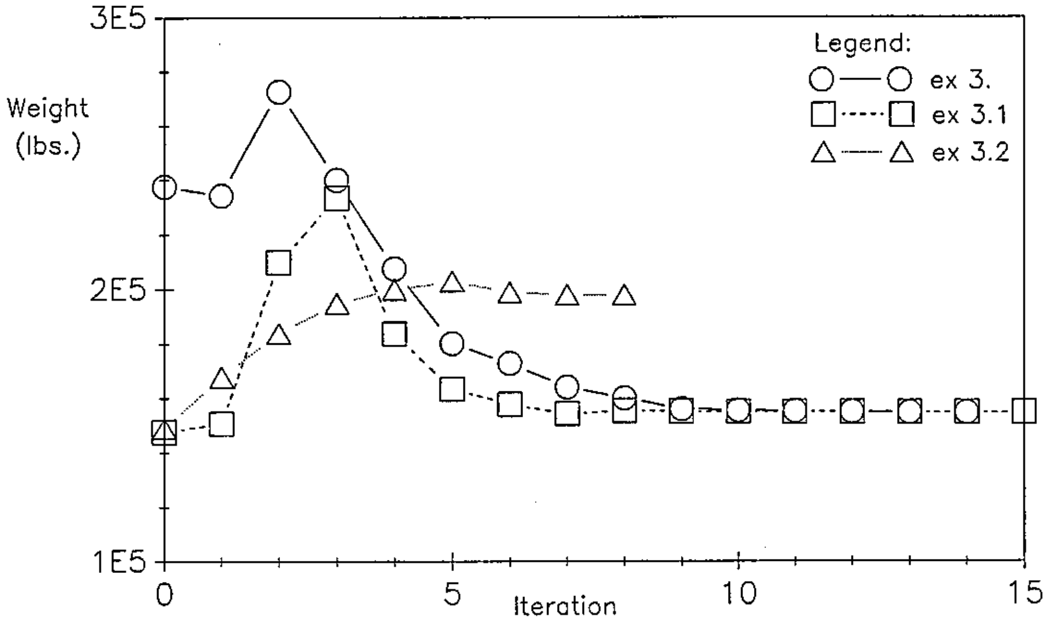


Figure 4–19. Convergence of weight for examples 3.–3.2.

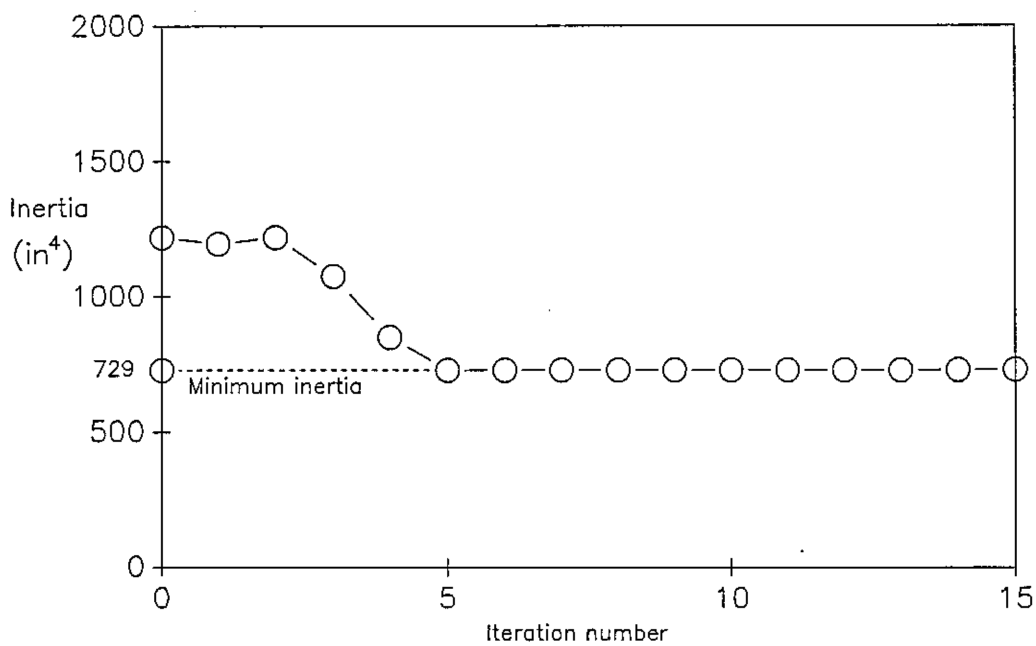


Figure 4–20. Example 3.2, convergence of size of group 3 to the minimum size.

<u>Example 3.1:</u> This example has a different starting point than example 3 as shown in Tables 4-14,17 and Figure 4-17. The initial cross-sections are much smaller. The resulting initial stress constraints indicate the design has constraint violations. In load case 2 all of the piles initially have a 41% stress constraint violation.

See the final layout in Table 4-18 and Figure 4-18. After 15 iterations the optimal point that is reached is almost exactly the same as in example 3. The weight originally increases due to the design starting in the infeasible region. See Figure 4-19. Example 3. and 3.1 converge to the same point. The optimum reached does not appear to be sensitive to the initial cross-section selection. This example also demonstrates that given a highly over-stressed initial design the feasible design space can be reached.

Table 4-17. Initial layouts and sizes for example 3.1

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	100.	90.	90.
2	729.	100.	90.	90.
3	729.	100.	90.	90.

Table 4-18. Final layouts and sizes for example 3.1

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	748.6	3.96	270.	90.
2	921.0	2.35	90.	90.
3	856.3	4.14	270.	90.

Example 3.2: The starting point of example 3. was varied so that the initial batters were opposite of their apparent optimal position as shown in Tables 4-14,19 and Figure 4-21.

See the final layout in Table 4-20 and Figure 4-22. The weight changes from 155839. to 198157. lbs in eight iterations. See Figure 4-19. The weight increases due to the infeasibility of the initial design. The active constraints are the minimum sizes for groups 1 and 3; the minimum batters for groups 1 and 3; and the stress in pile 11 for load case 2.

This example has a much worse final design than examples 3. and 3.1. This demonstrates that the design space does have local optimum points. Optimization algorithms have not been developed which can guarantee that the global optimum of a nonconvex problem can be found. A common method to search for the optimum is to use multiple initial designs. Two possible starting points exist for each batter. They are the opposite sides. An optimization algorithm can be developed to go through each of the possibilities. At this time it is expected that the experienced designer will select a few efficient initial layouts then choose the best final design produced.

Table 4-19. Initial layouts and sizes for example 3.2

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	3.	90.	90.
2	729.	3.	270.	90.
3	729.	3.	90.	90.

Table 4-20. Final layouts and sizes for example 3.2

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.0	1.00	90.	90.
2	785.2	2.20	270.	90.
3	729.0	1.00	90.	90.

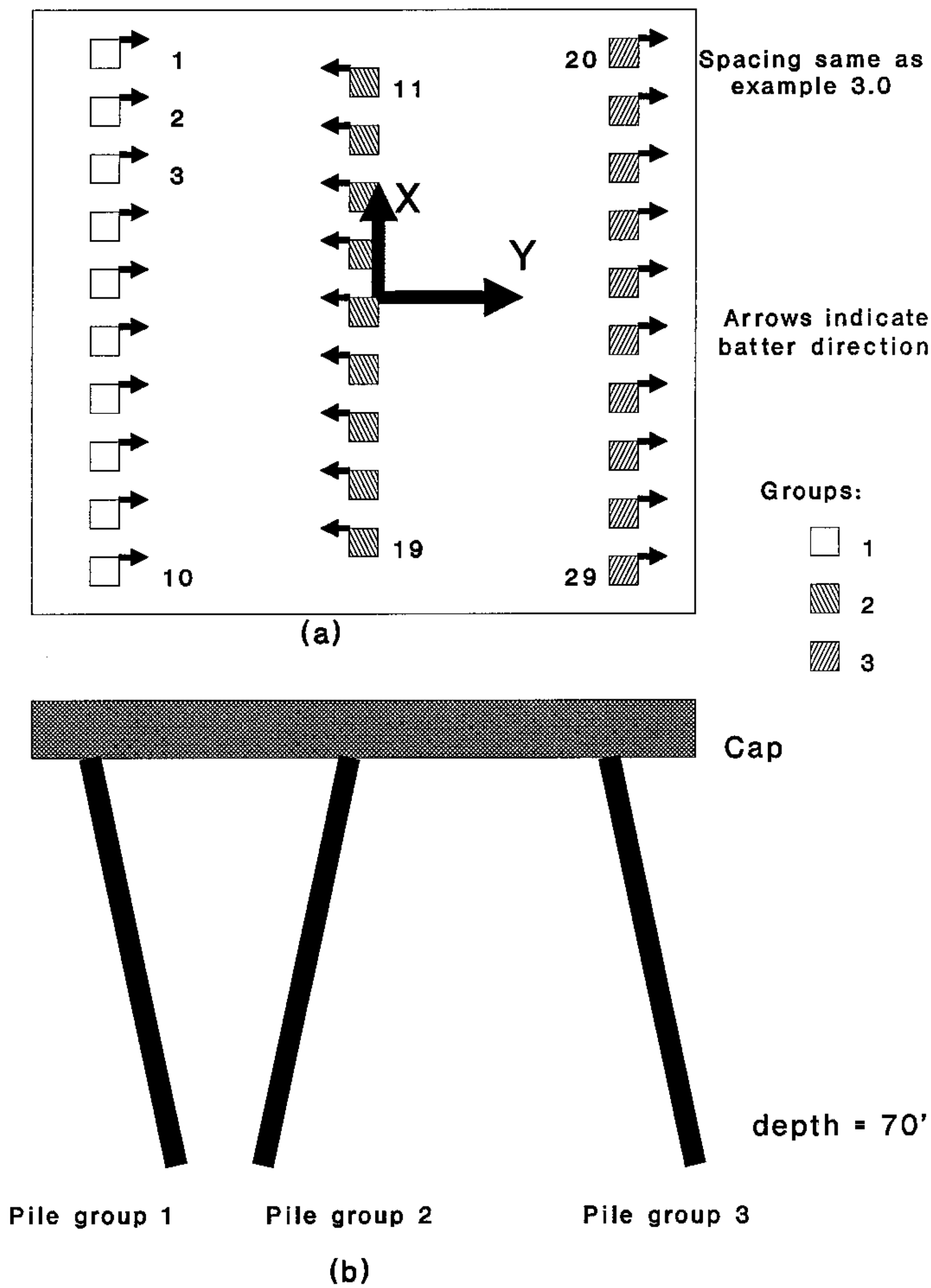


Figure 4-21. Initial pile layouts for example 3.2

(a) Plan view, (b) Elevation.

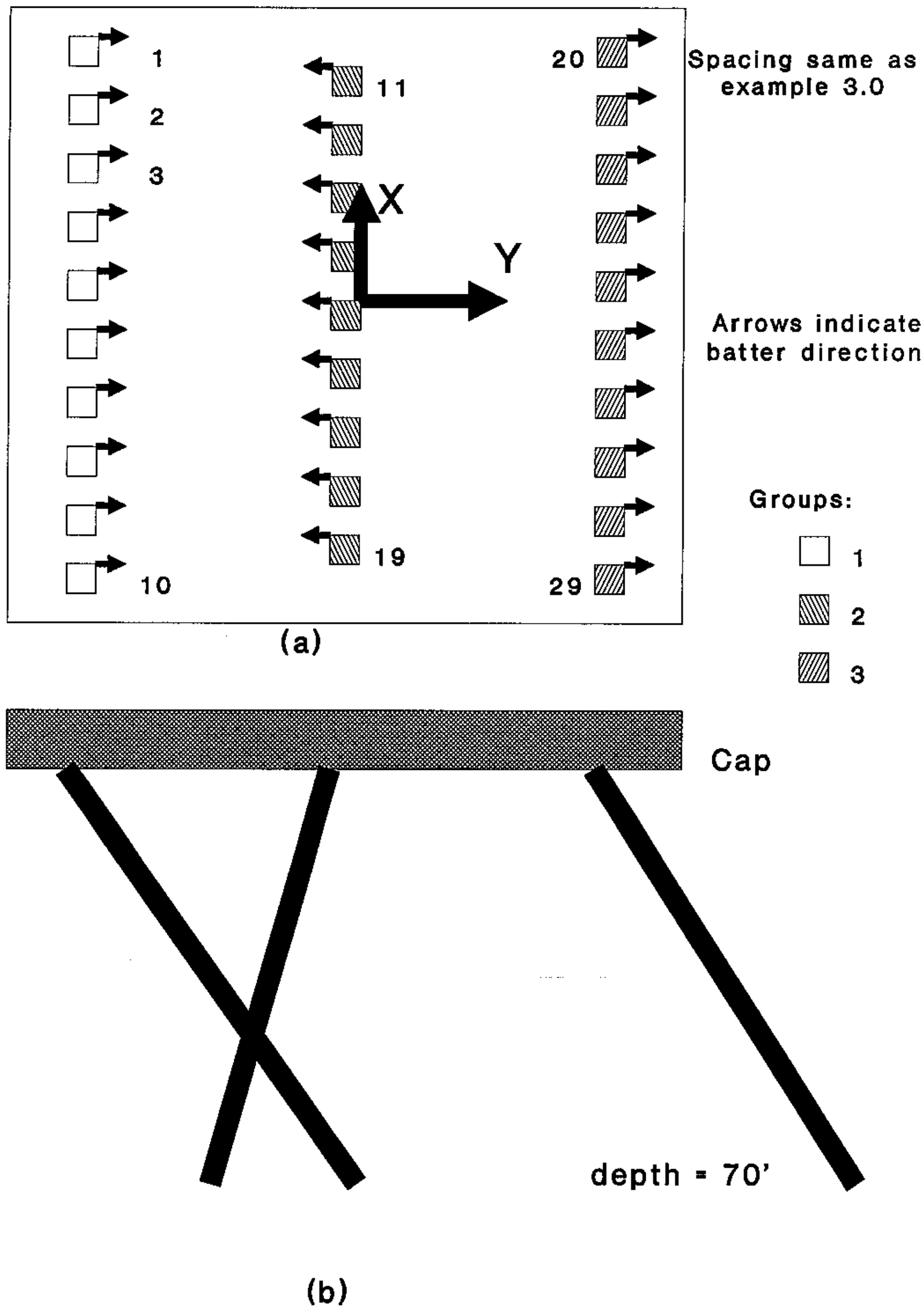


Figure 4-22. Final pile layouts for example 3.2

(a) Plan view, (b) Elevation.

4.4 EXAMPLE 4.

Example 4. This example has fifteen piles arranged into three linked groups as shown in Tables 4-21,22 and Figure 4-23. Piles 1 through 9 are linked into a group. The other groupings are: piles 10 through 12, and piles 13 through 15. Only the piles in the first group are aligned in the plane of the primary loading. This example is a pseudo three-dimensional optimization problem because the piles do not vary within the same planes. This example is not a fully three-dimensional because the angle phi is not a variable.

See the final layouts in Table 4-23 and Figure 4-24. The weight converges from 45120. to 39878. lbs in ten iterations as shown in Figure 4-25. The weight originally increases due to infeasibility of the initial design. The active constraints are the minimum sizes for groups 1 and 2; and the stresses in piles 7,10,13, and 13 for load cases 1,1,1 and 2 respectively.

Table 4-21. Loading for example 4.

Load case	P_x kips	P_y kips	P_z kips	M_x ft-k	M_y ft-k	M_z ft-k
1	-250.	0.	500.	200.	600.	0.
2	200.	300.	650.	0.	0.	0.

Table 4-22. Initial layouts and sizes for example 4.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	1029.	100.	0.	0.
2	1029.	10.	270.	0.
3	1029.	10.	90.	0.

Table 4-23. Final layouts and sizes for example 4.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.0	2.19	0.	0.
2	729.0	1.61	270.	0.
3	991.3	1.47	90.	0.

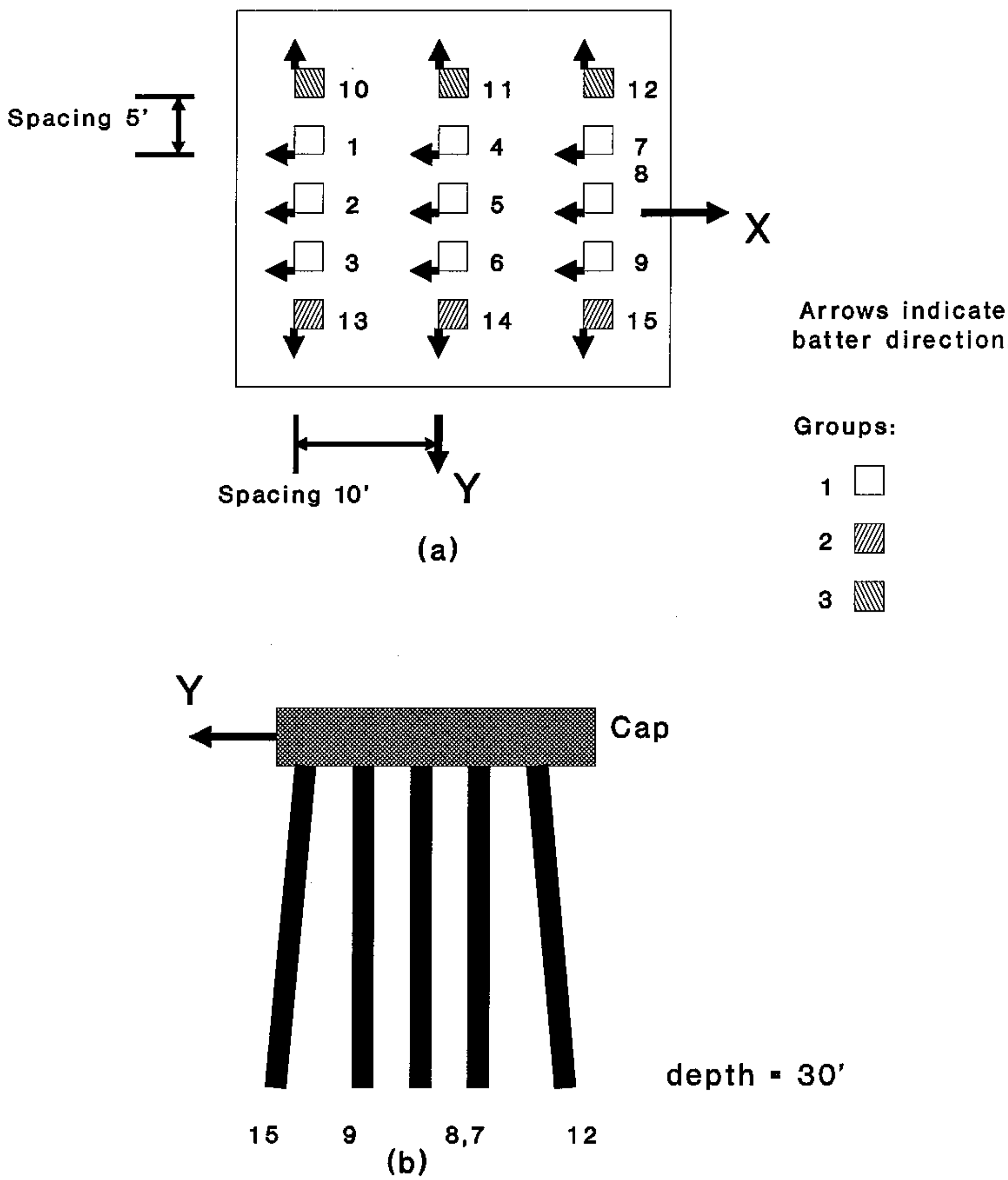


Figure 4-23. Initial pile layouts for example 4.
(a) Plan view, (b) Elevation.

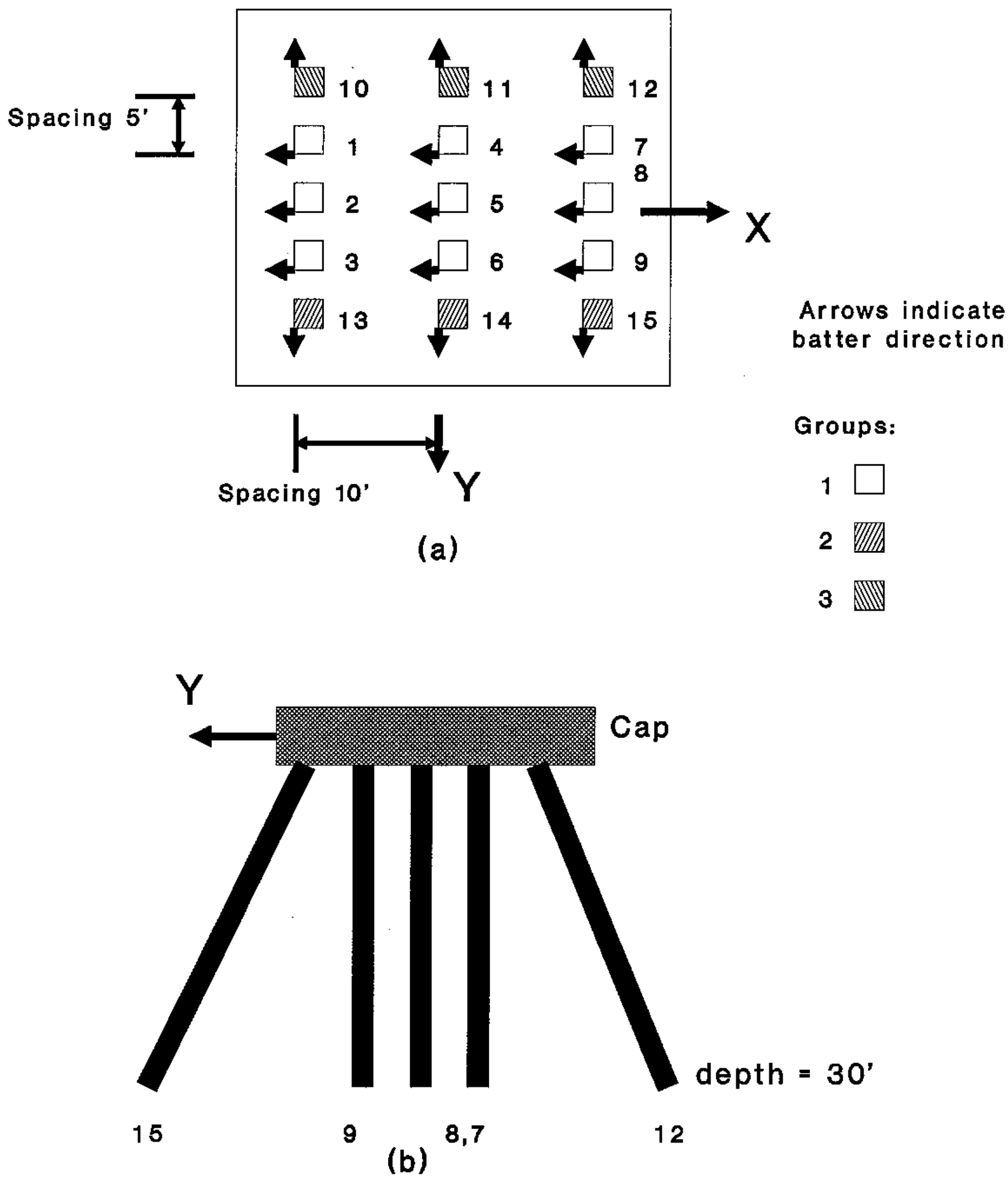


Figure 4-24. Final pile layouts for example 4.

(a) Plan view, (b) Elevation.

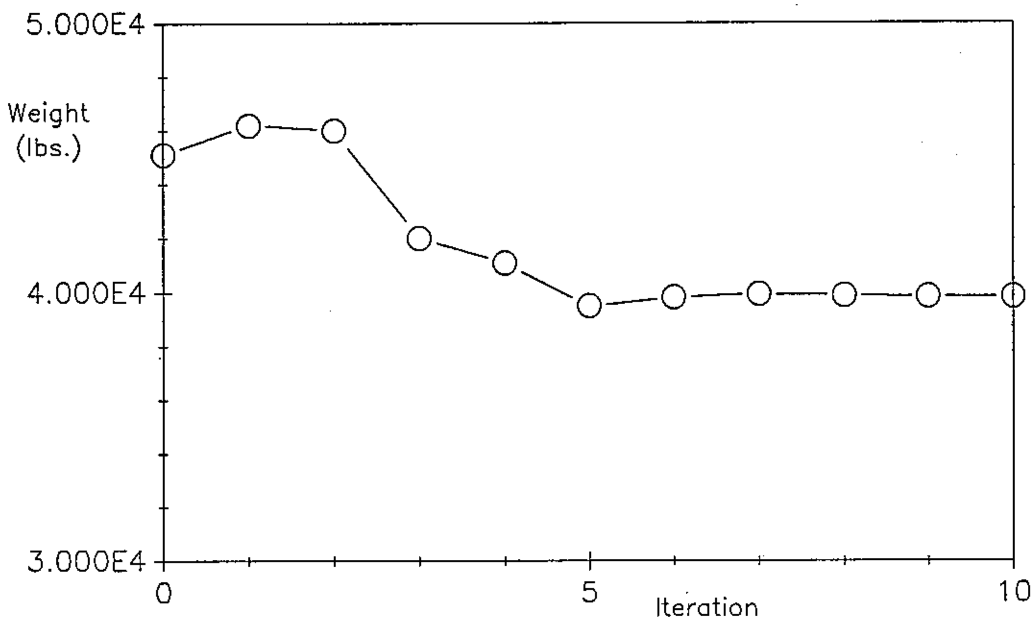


Figure 4–25. Convergence of weight for example 4.

4.5 EXAMPLE 5.

Example 5. This example has 240 pile arranged in three linked groups. This example is taken from Example 3 of the CPGA manual (9). The initial layouts are found in Tables 4-24,25 and Figure 4-26. The only alteration to the example is that all pile tips reach a depth of 90′. Note that the pile cap is sloped.

See the final layout in Table 4-26. The weight effectively converges after 8 iterations but the optimization algorithm is allowed to proceed for 15 iterations. The weight improves from 1477348. to 1435684. lbs as shown in Figure 4-27. The weight increases in the first iteration due to the linear representation of the weight gradients. The algorithm expected the weight to decrease. The linear gradients do not fully represent the design space. A non-beneficial iteration can be performed. If non-beneficial iterations occur then they usually occur early in the optimization process while the variables are changing the greatest. When the variables change greatly the actual weight has a greater chance of straying farther from the linearized prediction.

The active constraints are the minimum sizes for all groups; and the stress in pile 229.

Table 4-24. Loading for example 5.

Load case	P_x kips	P_y kips	P_z kips	M_x ft-k	M_y ft-k	M_z ft-k
1	0.	-12000.	20000.	-1.E6	0.	0.

Table 4-25. Initial layouts and sizes for example 5.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	100.	90.	90.
2	729.	2.	90.	90.
3	729.	2.	270.	90.

Table 4-26. Final layouts and sizes for example 5.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	6.58	270.	90.
2	729.	3.96	90.	90.
3	729.	3.92	270.	90.

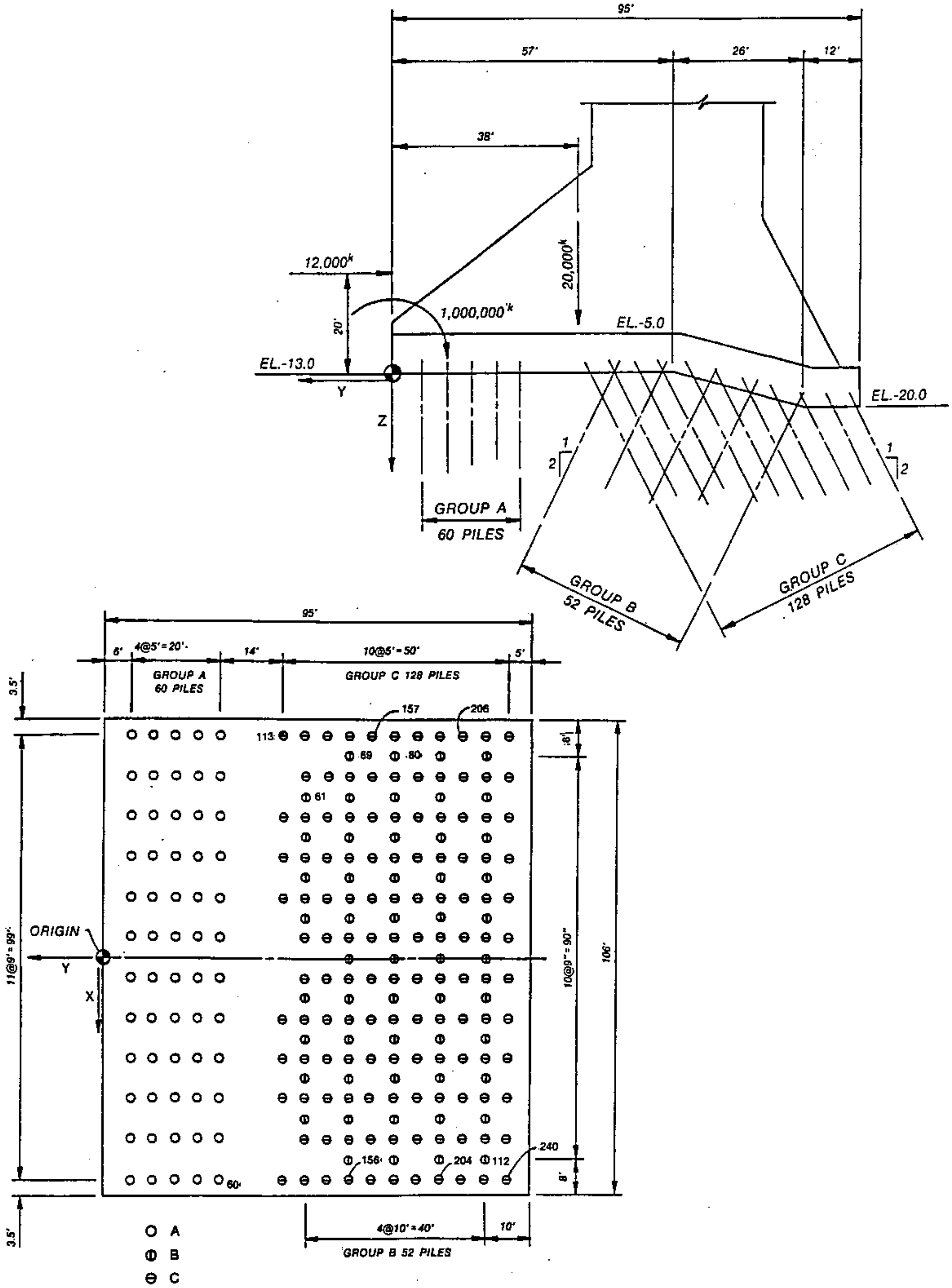


Figure 4-26. Initial Layout for example 5.
See CPGA manual (9) pp. B67,68.

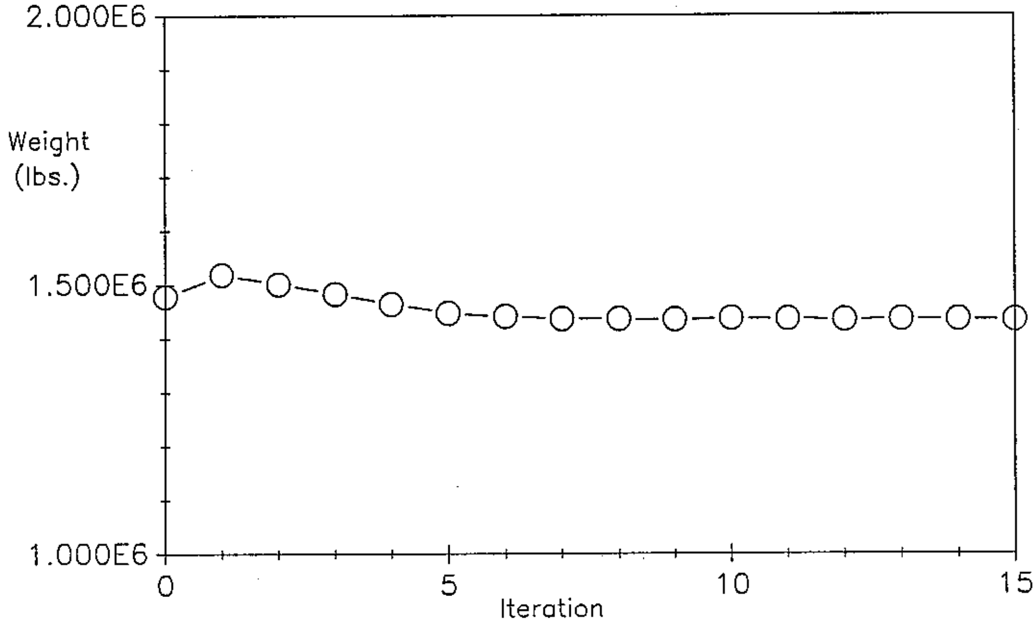


Figure 4–27. Convergence of Weight for example 5.

4.6 EXAMPLE 6.

Example 6. This example has 18 piles arranged in two groups. This example is taken from Example 5 of the CPGA manual (9). The only alterations are that the depth of piles 1-12 is held at 65′, and the depth of piles 13-18 is held at 74′. A displacement limit of 0.35 inches was arbitrarily added. See the initial layouts in Tables 4-27,28 and Figure 4-28. Note that the angle of orientation of each of the piles is not the same as the other piles within the group. This is a pseudo three-dimensional example.

See the final layout in Table 4-29. After seven iterations the weight converges from 95718. to 87728. lbs as shown in Figure 4-29. The active constraints are the minimum sizes of both groups; and the displacement component 1. See the displacement convergence in Figure 4-30.

Table 4-27. Loading for example 6.

Load case	P_x kips	P_y kips	P_z kips	M_x ft-k	M_y ft-k	M_z ft-k
1	-142.8	0.	1272.	0.	5600.	0.

Table 4-28. Initial layouts and sizes for examples 6., and 6.1.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	904.	4.	See Figure.	90.
2	729.	100.	See Figure.	90.

Table 4-29. Final layouts and sizes for example 6.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	729.	8.80	Unchanged.	90.
2	729.	14.81	Unchanged.	90.

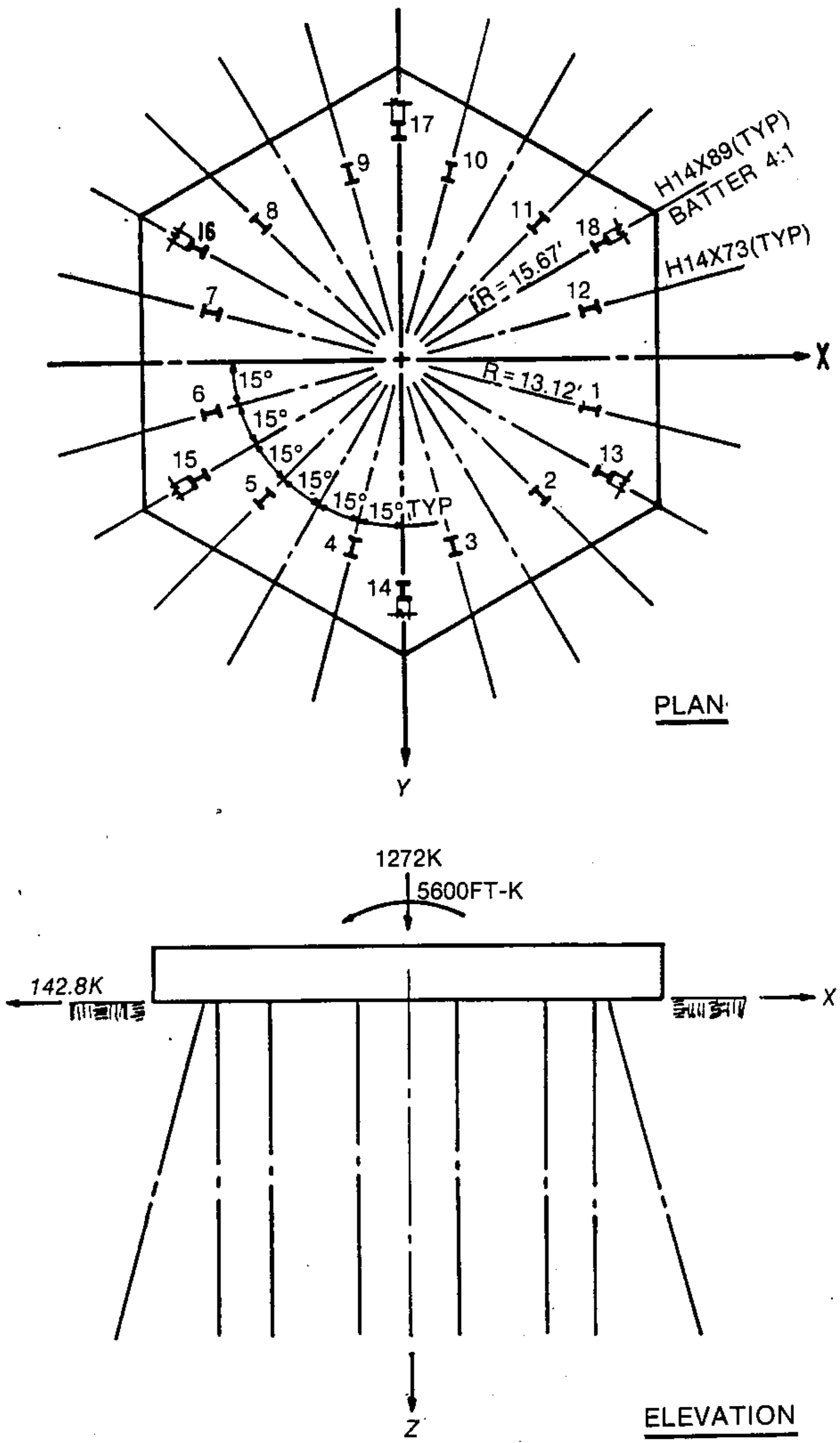


Figure 4-28. Initial Layout for example 6. and 6.1.
See CPGA manual (9) p. B125.

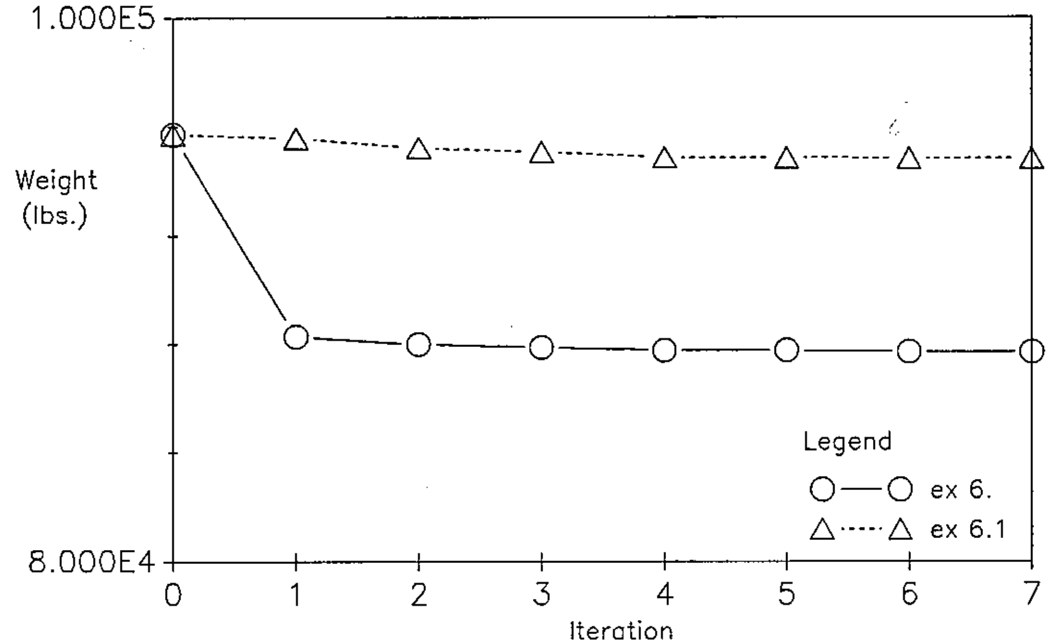


Figure 4–29. Convergence of weight for examples 6. and 6.1.

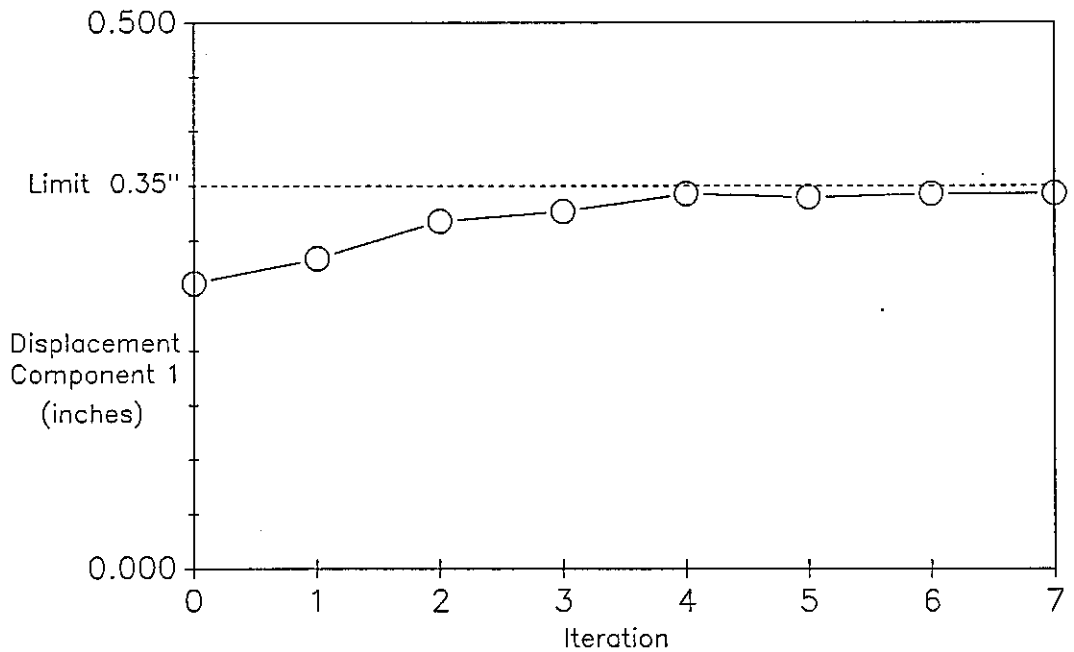


Figure 4–30. Satisfaction of displacement limit for example 6.

Example 6.1 This example has the same starting point as example 6. The moment of inertias are held constant to demonstrate the final phases of design. Discrete section sizes can be selected and held fixed. The example is then optimized with respect to the batters.

See the final layout in Table 4-30. The weight improves to 94830. after seven iterations as shown in Figure 4-29. Holding the inertias constant causes the final weight to be much higher than when they are allowed to vary. The only active constraint is the displacement component 1. See the convergence of the displacement component in Figure 4-31.

Table 4-30. Final layouts and sizes for example 6.1.

Pile Group	I_{xx} (in^4)	Batter	Phi (degrees)	Theta (degrees)
1	904.	9.13	Unchanged.	90.
2	729.	17.27	Unchanged.	90.

Table 4-31. Example soil type and fixity.

Example	Soil type	Pile Fixity
1	NH 0.019	All fixed
2	NH 0.019	All pinned
3	NH 0.019	All pinned
4	NH 0.019	All fixed
5	NH 0.019	All pinned
6	ES 0.312	All pinned

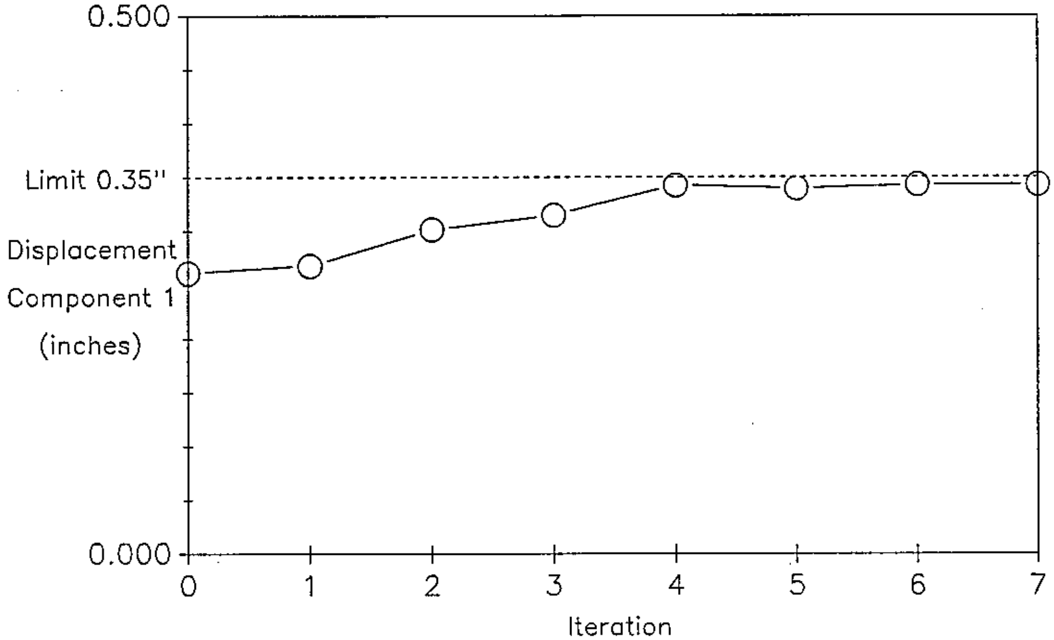


Figure 4–31. Satisfaction of displacement limit for example 6.1

5. SUMMARY AND CONCLUSIONS

5.1 CONCLUSIONS

An optimality criteria approach was adapted to structurally optimize the weight of steel HP-14 piles used to support structures which are modeled as rigid slabs. The examples included dams, a retaining wall, and a control tower. The optimization method is an iterative numerical process. An optimality approach was implemented because it converges quickly to an optimal solution.

The primary strategy of the optimality criteria approach is to form an optimality criteria by using the Lagrange equation for the weight and constraints. The optimality criteria is used to form a variable recurrence equation.

The secondary problem is to solve for the Lagrange Multipliers by anticipating which constraints are active and then solving a set of linear equations. The algorithm presented for selection of active constraints has worked for all examples, and for all other feasible problems that were attempted. The selection process uses a large portion of the computation time therefore this is an area of potential research.

A method was developed to promote the best convergence of topological variables with optimality criteria. The coordinate system for each topological variable is selected according to the predictability of the variable, and according to how the weight changes with the variable. Since topological variables can have a negative weight gradient, the optimality criteria methods were adapted to properly handle such variables by altering the coordinate systems.

An alternative energy objective function is used for a variable when the variable has no direct effect on the weight. A weight gradient is required for all variables with the current optimality criteria version. Energy is chosen to be minimized because it promotes lower displacements which are generally desirable.

The pile cross-sections and pile batters were allowed to vary. The variables were not held to discrete values but were allowed to change in a continuous manner. The pile cross-sections were represented by a single primary variable which was the section's major axis inertia. Equations were developed to closely approximate the other cross-section variables such as area given the inertia.

Six primary examples were optimized. The number of piles in each of the examples ranged from 5 to 240 piles. The piles were either placed in linked groups or they were allowed to independently vary. Linking piles into groups provides consistency that is desired when installing the piles, and linking eases computational requirements.

The designs which had initial constraint violations quickly approached the feasible design space. The initially feasible problems had slight to significant weight improvement from the initial design. The number of iterations for the designs to converge was relatively small. In a few iterations the design was usually within the neighborhood of the final converged design.

As a few of the designs approached the converged design the variables and weight oscillated. The estimate of the optimum point using the linearizations of the constraints and weight function can overshoot or undershoot the optimum. Overshooting and the resulting oscillation can be reduced by increasing the convergence control parameter. The linearizations also caused Example 5 to have a non-beneficial iteration. The linear algorithm expected a weight improvement in the first iteration even though it did not occur.

Locally optimal points with weights greater than the global optimum may be reached. It was demonstrated that the optimization algorithm can be sensitive to the choice of the initial batters, but it does not appear as sensitive to the choice of initial cross-sections. The experienced designer should use multiple starting points for the optimization process. The choice of which piles to link will have an effect on the final optimal solution. The designer may choose to leave the piles unlinked until the optimum is approached.

5.2 AREAS FOR FURTHER RESEARCH

The analysis method used considers the pile cap to be a rigid slab. When the pile cap is considered to be flexible then a different optimum point may be found. A flexible cap analysis program will be altered with optimization subroutines to find optimal designs.

The optimization algorithm handles the cross-sections as a continuous variable. A discrete optimization method should be used to find a optimal discrete cross-sectional size. The elimination of unnecessary piles would provide large weight improvements. It may also be treated as a discrete process.

The angle of pile rotation about the vertical axis was not a variable in the optimization process. This angle should be a variable for these examples to be completely three-dimensional. Examples 2 through 6 were pseudo three-dimensional because the piles were not contained within a plane. The pile angles were not required to be constant within a group.

The pile tip interference constraints are very complicated in three-dimensions. A simplified expression for these constraints will be found. The usage of energy as an objective function also will be made more sophisticated to aide convergence of the angle variables. Other variables that should be added to the optimization algorithm are pile cap fixity, and relation of the pile's major axis to the batter direction. Both of these variables would have a discrete nature.

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7. APPENDIX: PILE GROUP BEHAVIOR

The optimization methods are applied to an existing pile group analysis computer program. The analysis methods are not altered by the optimization algorithm. The following is a discussion of the pile group analysis methods which is taken from BASIC PILE GROUP BEHAVIOR (10), pp. 6,7.

The "basic method of pile group analysis . . . is valid for static analysis of a linear, elastic system. Interaction between pile and structure is limited to the extremes of a fully fixed or fully pinned connection. Interaction between the pile and soil is represented by a linear, elastic pile stiffness (applied load per unit deflection) at the top of the pile. The base of the structure is assumed to act as a rigid body pile cap connecting all piles; the cap flexibility is not considered. This method of analysis will also be incorporated in the new pile group analysis program.

"The basic pile group analysis method represents each pile by its calculated stiffness coefficient, in the manner proposed by Saul. The stiffness coefficients of all piles are summed to determine a stiffness matrix for the total pile group. Displacements of the rigid pile cap are determined by multiplying the sets of applied loads by the inverse of the group stiffness coefficients to determine the forces acting on each pile head. The key step in the method is in determining individual pile stiffness coefficients, at the pile head, based on known or assumed properties of pile and soil. Since this is a three-dimensional analysis method, each pile head has six degrees of freedom (DOF), three translations and three rotations. A stiffness coefficient must be determined for each DOF and for all coupling effects (e.g. lateral deflection due to applied moment). The pile location and batter angle are also accounted for when individual pile stiffness coefficients are combined to form the total stiffness matrix for the pile group.

"Piles are mathematically represented in the analysis by their axial, lateral and rotational stiffness, as springs resisting motion of the rigid cap."

The pile stresses are evaluated to determine the feasibility of the design by use of interaction equations.

The following equations are taken from BASIC PILE GROUP BEHAVIOR (10), pp. A8,A9.

"Steel piles subject to axial load and bending shall be proportioned to satisfy the following requirements:

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{by}}{(1-f_a/F'_{ex})\,F_{bx}} + \frac{C_{my} f_{by}}{(1-f_a/F'_{ey})\,F_{by}} \leq 1.0$$

and:

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \quad (when \frac{f_a}{F_a} \leq 0.15)$$

where:

$$F'_e = \frac{\pi^2 E}{F.S.\ (K_b L_b / r_b)^2}$$

and:

f_a =	computed axial stress (psi)
f_{bx} or f_{by} =	computed compressive bending stress about the x axis and y axis, respectively (psi)
F_a =	allowable axial stress (psi)
F_{bx} or F_{by} =	allowable compressive bending stess about the x and y axis, respectively (psi)
E =	modulus of elasticity (29,000,000 psi)
L_b =	actual unbraced length of pile in the plane of bending (inches)
K_b =	effective length factor as defined by AISC in the plane of bending (inches)
r_b =	radius of gyration in the plane of bending (inches)
C_{mx} or C_{my} =	coefficient about x and y axes, respectively, as defined by AISC
F.S. =	Factor of Safety

The lower regions of the bearing piles are subject to axial stresses. Since the lower region of the pile is subject to damage during driving a greater factor of safety is used. The axial load at the tip must be less than the damaged cross-section capacity and the soil bearing capacity.

The point of maximum moment in a pile is dependant upon the soil conditions and the fixity of the pile to the cap. The following is taken from the CPGA Manual (9), pp. 44,45.

"The pile forces, used for the above comparisons with the allowables, are those forces calculated as acting on the top of the pile. "Pinned" piles, however, have no moment at the top, but do experience bending beneath the top, depending on lateral loads and soil properties. The critical moments may be approximated for design purposes as a constant times the lateral force on the pile

M1 = KMP1 F2

where

M1 = design moment
F2 = lateral force
KMP1 = constant

"These calculated moments are then used in the allowable load comparisons. When pile and soil properties have been given for a pinned pile, the design moment factors, KMP1 and KMP2, may be calculate automatically."

8. NOMENCLATURE

W_T = total weight of the steel piles.

γ = density of steel.

V_i = volume of element i.

N = number of piles.

h_j = constraint j.

m = number of constraints.

σ_j = stress in member j.

$\bar{\sigma}_j$ = upper limit on the stress in member j.

I_{xx} = major axis moment of inertia.

$\underline{Ixx}$ = lower bound on the major axis moment of inertia.

$\bar{I}_x$ = upper bound on the major axis moment of inertia.

L = Lagrangian.

λ_j = Lagrange Multiplier for the j^{th} constraint.

d_i = design variable number i.

n = number of variables.

r = convergence control parameter.

n_1 = number of active variables.

I_{yy} = minor axis moment of inertia of a given member.

A = area of a given member.

c_x = extreme fiber distance in the direction of a member's local x axis.

c_y = extreme fiber distance in the y direction.

P_i = global force component i.

D_i = global displacement component i.

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