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# Optimum roof angles of steel gable frames with pinned supports

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**--------------------------------------------------------ABSTRACT--------------------------------------------------------**
Optimum roof slopes were found for steel gable frames with pinned supports. Five locations, Detroit MI, Miami FL, Santa Barbara CA, St. Paul MN and Berlin NH, were chosen because they represented a variety of loading conditions. Multiple structures were designed for each location with span to height ratios from 1 to 10, and roof angles from 10 to 45 degrees. Analysis was done for each case using ETABS. This provided patterns in stress efficiencies that the helped identify possible optimum angles. Precise designs with the LRFD method were performed on each roof angle for each span length if it was a candidate for the optimum. It was found that for low snow load areas, a low roof angle is optimum. However, as roof load and span increase, the optimum roof angle is higher. A primary factor in determining roof angles in these situations was having an angle high enough so that unbalanced snow loading was not required by the code.

**KEYWORDS:** Gable frame design, Pre-fabricated frames, Portal steel frames, optimization, Pitch roof steel frame and roof slope.

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## I. INTRODUCTION

The purpose of designing a structure is that an experienced engineer uses methods of art and science to find a safe and economic or optimum solution. To reach an optimum design, several criteria need considered to make sure the best solution has been reached. Some of the economic criteria are minimum usage of material, minimum time of fabricating and constriction, minimum number of workers and their related costs. Minimizing material generally reduces other factors such as construction time, and so consequently reduces the overall cost of the structure and construction. To get lowest possible amount of material, the design engineers must be aware of all factors that could control the design. One factor in efficient design is laying out the structure so that it has proportional geometry. Examples are choosing frame and column spacing and roof slope so that they are proportionate and match local conditions. Choosing inappropriate geometry like roof slope could waste material and increase overall construction costs by increasing material usage, fabricating time, and labor costs. Previously, it has been shown that the LRFD and ASD can produce significantly different designs of steel gable frames [1]. LRFD and ASD are based on different philosophies of design and often don't produce the same resulting design. In general, LRFD produces more economical gable designs or is nearly the same as ASD. There are few situations where ASD produces significantly lighter weight steel gable frame designs. Additionally, it has been shown that in high snow regions, the difference between the methods is more dramatic [2].

The purpose of this study is to find the optimum roof slope for steel gable frames. If generalizations can be found, then designers could have a guideline for determining the initial geometry of gable frames. The choice of the optimization method depends upon factors such as whether the problem in non-linear with local minima and maxima, whether the variables are discrete or continuous, and whether the process can easily be automated through computer formulation. If the problem has little risk of local minima and maxima, the variables are continuous, and if the design can be automated, then a set of optimization methods are suitable such as linear programming or optimality criteria [3].Some structural variables are limited to discrete values. For example, steel plates are only available with specific thicknesses. In those situations, using continuous optimization variables can still sometimes used if a branch and bound process is applied [4]. This works efficiently as long as the problem isn't highly non-linear. If a problem is highly non-linear or has discrete variables that be made pseudo continuous, then other methods could be used such as a genetic optimization [5]. With this method, random initial starting points are made, and then the attributes of the best designs are combined in new permutations. Saka [6] found that genetic Programming for gables had only a few design variables, but the research focused only on one type of loading, and it designed haunches that were welded to structural shapes. Its goal was development of a method rather than application to practical situations.In summary, the optimization

Problem turned out to be not very complex. The steel cross-sectional properties could have been optimized, and even though the variables are discrete, such as plate size, they could have been made continuous. Instead, the plates were chosen based on practical engineering judgment. The cross-sectional depth varied linearly along the length, but it was not practical to vary the other plate dimensions the same way. Additionally, only a small set of possible roof angles needed to be checked. Therefore, the optimization method that was used was a global search where every possible solution is checked except for solutions that are obviously not the optimum.

## II. METHODS

Snow, wind and earthquake are the loads on gable frames that vary by location. We wanted a case to illustrate when each of the three situations was high. We sought extra locations with varying snow since it was seen previously that gable frame designs were sensitive to snow levels [2]. Therefore, it was determined that five different locations would be necessary to illustrate how variation in commonly controlling loads influences which roof slope would be optimum for pre-fabricated gable frames. See Table 1 for the list of the locations and loads used. Seismic load was not controlling (NC) for most locations.

**Table 1. Locations and Loads Used**

| **Location** | **Dead Load (Psf)** | **Roof Live (psf)** | **Ground Snow (psf)** | **Wind Speed (mph)** | **Earthquake** | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | $S_s$ | $S_1$ | $F_a$ | $F_v$ |
| **Detroit, MI** | 20 | 20 | 20 | 90 | NC | NC | NC | NC |
| **St. Paul, MN** | 20 | 20 | 50 | 115 | NC | NC | NC | NC |
| **Berlin, NH** | 20 | 20 | 100 | 108 | NC | NC | NC | NC |
| **Miami, FL** | 20 | 20 | 0.00 | 170 | NC | NC | NC | NC |
| **Santa Barbara, CA** | 20 | 20 | 0.00 | 93 | 2.19g | 0.79g | 1 | 1.7 |

The column height and space between frames were a constant 20 feet for all cases. The gable spans were 20, 30, 40, 50, 60, 70, 80, 90, 100, 150, and 200 feet because wind load changes with the ratio of span to height and span to width. Roof angles were varied between 10, to 45 degrees because of changes in the wind and snow loads with the roof angle. The column base support was hinged. For determining the unbraced length of the compression flange, the space between purlins was assumed to be 3 feet. See the frame geometry in Figures 1 and 2. One typical frame in the middle was designed and is shaded in Figure 2. The total width of the structure needed to be set at a constant value for consistency. This building width influences the calculation of the leeward side wind load. There was no particular reason to pick one width over another so 20' was chosen which the width of only one segment is. A check found [1] that the result only varied by about 1% when the width was changed, so in most cases the results are not significantly dependent on it.

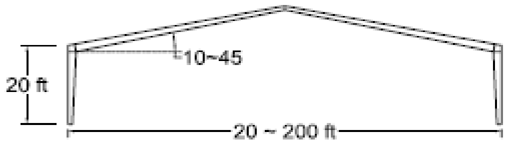


**Figure 1 Typical Frame**

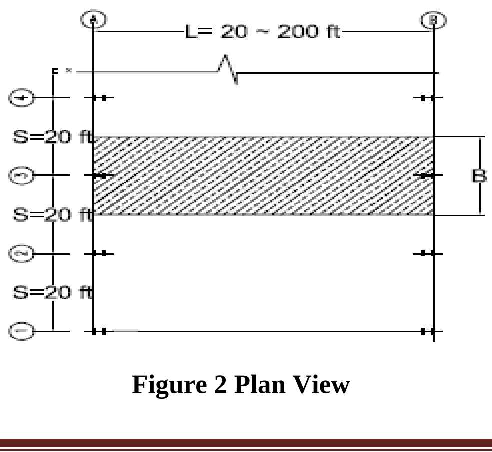


**Figure 2 Plan View**

To follow common fabrication practice, the column and rafter were defined as non-prismatic member. Webs are assumed be linearly tapered and flanges are assumed be constant. Therefore, the major axis moment of inertia will vary non-linearly in the column and rafter. Since the optimization method is a global search, the initial designs for each location, span ratio and roof angle would be analyzed. The same initial member sizes would be used for each case as shown in Table 2.The stress utilization of the initial member will be used to determine is a roof angle is a candidate for being the optimum angle. If that angle is a candidate, then it will be designed. Depending upon the loading and span, the initial member might be overstressed or under-stressed. The weight is related to the cross-sectional dimensions chosen, but also is a function of the length of a member. A higher roof angle requires more material because the members are longer as they angle more steeply. Therefore, if the smallest angle has the lowest stress utilization, then it is obviously the best angle for the design, and it is unnecessary to find the precise designs for the other angles. The members were designed to keep stress ratio in the members close to 1 and satisfy lateral frame displacement limit.

**Table 2. General section for part one of study**

| Position | θ | L/H | Member | Flange | | Web | |
|---|---|---|---|---|---|---|---|
| | | | | b | t | h | t |
| Base | 10~45 | 1~10 | Column | 15 | 1 | 14 | 0.375 |
| Eave | | | | 15 | 1 | 30 | 0.375 |
| Eave | | | Rafter | 15 | 1 | 30 | 0.375 |
| Ridge | | | | 15 | 1 | 14 | 0.375 |

The moment frame considered was as an ordinary moment frame. The site class and seismic design category were assumed to be D. The importance factor was taken at 1. The surface roughness category was considered exposure C. The roof slope condition assumed was an unobstructed slippery surface and considered as a warm roof. For wind loading, the directional procedure was used from ASCE 7-16 [7]. Site Class D is used for each city so that comparisons between locations can be made. However, conditions at actual project sites may vary from the hypothetical. Additionally, Exposure C was used for wind, but that doesn't mean the predominant exposure in the area is that type. The direct method was used in frame analysis. Moments in columns are expected to govern since the frames do not have significant axial loads from cranes or other attachments. Consequentially, axial analysis and determining the k factor bear little on the final result. The ASTM standard A572 high- strength steel, grade 50 has been used for design members ($F_y = 50\ ksi$ , $F_u = 65\ ksi, F_{ye} = 55\ ksi, F_{ue} = 71.5\ ksi, E = 29{,}000\ ksi$).
For designing members AISC 360-16 was used [8]. Analysis and design has been done by ETABS 17.

## III. RESULTS

Table 3 to 7 show the results for each city. In Table 3, for Detroit, MI, the lowest roof angles have most of the lowest stress utilizations, so it was not necessary to calculate the weights of each design. (Entries that were not calculated are shown with N.C.) The benefit of this research is not to determine exact weights but to find the optimum roof angles. However, in Table 4, for Berlin, NH, the high snow load made the optimum roof angles higher. Therefore, it was necessary to do more of the designs and weight calculations.

**Table 3-Optimum Roof Slope for Detroit MI.**

| L/H | Out put | θ = 10 | θ = 15 | θ = 20 | θ = 25 | θ = 30 | θ = 35 | θ = 45 | Optimum θ (Degree) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Ave Stress Ratio | 0.048 | 0.050 | 0.058 | 0.063 | 0.070 | 0.075 | 0.089 | 10 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 1.5 | Ave Stress Ratio | 0.059 | 0.062 | 0.073 | 0.082 | 0.091 | 0.099 | 0.121 | 10 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 2 | Ave Stress Ratio | 0.088 | 0.091 | 0.105 | 0.111 | 0.117 | 0.128 | 0.158 | 10 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 2.5 | Ave Stress Ratio | 0.129 | 0.129 | 0.148 | 0.155 | 0.160 | 0.168 | 0.200 | 10 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 3 | Ave Stress Ratio | 0.178 | 0.177 | 0.198 | 0.205 | 0.208 | 0.217 | 0.245 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 3.5 | Ave Stress Ratio | 0.234 | 0.231 | 0.254 | 0.259 | 0.261 | 0.270 | 0.295 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 4 | Ave Stress Ratio | 0.298 | 0.291 | 0.316 | 0.318 | 0.317 | 0.325 | 0.352 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 4.5 | Ave Stress Ratio | 0.369 | 0.356 | 0.382 | 0.380 | 0.376 | 0.384 | 0.410 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 5 | Ave Stress Ratio | 0.446 | 0.426 | 0.451 | 0.445 | 0.438 | 0.444 | 0.471 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 7.5 | Ave Stress Ratio | 0.905 | 0.822 | 0.837 | 0.799 | 0.765 | 0.757 | 0.762 | 15 |
| | Weight (lbs.) | 19552 | 18093 | N.C | N.C | N.C | 19603 | N.C | |
| 10 | Ave Stress Ratio | 1.464 | 1.283 | 1.295 | 1.189 | 1.120 | 1.092 | 1.070 | 15 |
| | Weight (lbs.) | 34671 | 28980 | 29945 | 30043 | 31071 | 32825 | 38392 | |

**Table 4-Optimum Roof Slope for Berlin, NH**

| L/H | Out put | θ = 10 | θ = 15 | θ = 20 | θ = 25 | θ = 30 | θ = 35 | θ = 45 | Optimum θ (Degree) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Ave Stress Ratio | 0.059 | 0.059 | 0.062 | 0.065 | 0.067 | 0.071 | 0.083 | 10 |
| | Weight (lbs.) | 1338 | 1422 | 1453 | 1472 | 1501 | 1529 | 1623 | |
| 1.5 | Ave Stress Ratio | 0.11 | 0.112 | 0.114 | 0.117 | 0.12 | 0.118 | 0.125 | 10 |
| | Weight (lbs.) | 1821 | 1897 | 1934 | 1976 | 2034 | 2112 | 2234 | |
| 2 | Ave Stress Ratio | 0.1665 | 0.168 | 0.1645 | 0.1645 | 0.1605 | 0.132 | 0.1515 | 30.3 |
| | Weight (lbs.) | 3095 | 3105 | 3134 | 3178 | 2813 | 2965 | 3109 | |
| 2.5 | Ave Stress Ratio | 0.311 | 0.292 | 0.272 | 0.256 | 0.238 | 0.24 | 0.245 | 30.3 |
| | Weight (lbs.) | 4234 | 4123 | 4022 | 3998 | 3876 | 4011 | N.C | |
| 3 | Ave Stress Ratio | 0.367 | 0.347 | 0.331 | 0.321 | 0.303 | 0.216 | 0.24 | 30.3 |
| | Weight (lbs.) | 5506 | 5398 | 5103 | 4876 | 4500 | 4681 | N.C | |
| 3.5 | Ave Stress Ratio | 0.565 | 0.522 | 0.479 | 0.45 | 0.412 | 0.386 | 0.39 | 30.3 |
| | Weight (lbs.) | 7856 | 7645 | 7489 | 7323 | 6987 | 7108 | N.C | |
| 4 | Ave Stress Ratio | 0.626 | 0.578 | 0.5345 | 0.509 | 0.4705 | 0.3175 | 0.338 | 30.3 |
| | Weight (lbs.) | 10421 | 9305 | 8456 | 7567 | 6339 | 6872 | N.C | |
| 4.5 | Ave Stress Ratio | 0.89 | 0.818 | 0.739 | 0.674 | 0.607 | 0.549 | 0.534 | 30.3 |
| | Weight (lbs.) | 12876 | 11987 | 10880 | 9989 | 9234 | 9789 | N.C | |
| 5 | Ave Stress Ratio | 0.94 | 0.85 | 0.776 | 0.719 | 0.655 | 0.433 | 0.45 | 30.3 |
| | Weight (lbs.) | 15502 | 15380 | 13456 | 11567 | 9093 | 9552 | N.C | |
| 7.5 | Ave Stress Ratio | 1.7005 | 1.4815 | 1.3125 | 1.184 | 1.054 | 0.681 | 0.684 | 30.3 |
| | Weight (lbs.) | 33753 | 29440 | 23654 | 19850 | 16912 | 18397 | N.C | |
| 10 | Ave Stress Ratio | 2.938 | 2.525 | 2.207 | 1.9385 | 1.688 | 1.076 | 1.049 | 30.3 |
| | Weight (lbs.) | 68620 | 63448 | 53752 | 44675 | 35540 | 37450 | N.C | |

**Table 5-Optimum Roof Slope for St. Paul, MN**

| L/H | Out put | θ = 10 | θ = 15 | θ = 20 | θ = 25 | θ = 30 | θ = 35 | θ = 45 | Optimum θ (Degree) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Ave Stress Ratio | 0.049 | 0.052 | 0.059 | 0.064 | 0.070 | 0.075 | 0.089 | 10 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 1.5 | Ave Stress Ratio | 0.067 | 0.067 | 0.078 | 0.084 | 0.092 | 0.099 | 0.121 | 10 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 2 | Ave Stress Ratio | 0.104 | 0.100 | 0.114 | 0.117 | 0.118 | 0.128 | 0.158 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 2.5 | Ave Stress Ratio | 0.155 | 0.146 | 0.163 | 0.165 | 0.162 | 0.168 | 0.200 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 3 | Ave Stress Ratio | 0.216 | 0.202 | 0.220 | 0.220 | 0.213 | 0.217 | 0.244 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 3.5 | Ave Stress Ratio | 0.289 | 0.265 | 0.284 | 0.280 | 0.268 | 0.270 | 0.294 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 4 | Ave Stress Ratio | 0.370 | 0.335 | 0.351 | 0.341 | 0.324 | 0.325 | 0.350 | 30.3 |
| | Weight (lbs.) | 7050 | 6986 | N.C | N.C | 6830 | 7271 | N.C | |
| 4.5 | Ave Stress Ratio | 0.460 | 0.413 | 0.426 | 0.410 | 0.387 | 0.384 | 0.407 | 30.3 |
| | Weight (lbs.) | 9620 | 8237 | N.C | N.C | 7678 | 7910 | N.C | |
| 5 | Ave Stress Ratio | 0.557 | 0.496 | 0.507 | 0.478 | 0.447 | 0.444 | 0.466 | 30.3 |
| | Weight (lbs.) | 10905 | 9046 | N.C | N.C | 8737 | N.C | N.C | |
| 7.5 | Ave Stress Ratio | 1.137 | 0.960 | 0.944 | 0.865 | 0.782 | 0.757 | 0.762 | 30.3 |
| | Weight (lbs.) | 25960 | 19903 | 19482 | 18750 | 17580 | 18757 | N.C | |
| 10 | Ave Stress Ratio | 1.840 | 1.505 | 1.425 | 1.280 | 1.143 | 1.092 | 1.074 | 30.3 |
| | Weight (lbs.) | 46590 | 36600 | 33700 | 30873 | 29034 | 32370 | N.C | |

**Table 6-Optimum Roof Slope for Miami, FL**

| L/H | Out put | θ = 10 | θ = 15 | θ = 20 | θ = 25 | θ = 30 | θ = 35 | θ = 45 | Optimum θ (Degree) |
|---|---|---|---|---|---|---|---|---|---|
| **1** | **Ave Stress Ratio** | 0.092 | 0.096 | 0.109 | 0.120 | 0.135 | 0.144 | 0.174 | 10 |
| | **Weight (lbs.)** | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| **1.5** | **Ave Stress Ratio** | 0.098 | 0.104 | 0.123 | 0.140 | 0.159 | 0.174 | 0.218 | 10 |
| | **Weight (lbs.)** | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| **2** | **Ave Stress Ratio** | 0.109 | 0.116 | 0.143 | 0.168 | 0.188 | 0.212 | 0.272 | 10 |
| | **Weight (lbs.)** | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| **2.5** | **Ave Stress Ratio** | 0.143 | 0.146 | 0.172 | 0.211 | 0.225 | 0.258 | 0.334 | 10 |
| | **Weight (lbs.)** | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| **3** | **Ave Stress Ratio** | 0.189 | 0.193 | 0.223 | 0.244 | 0.267 | 0.308 | 0.401 | 10 |
| | **Weight (lbs.)** | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| **3.5** | **Ave Stress Ratio** | 0.243 | 0.245 | 0.280 | 0.297 | 0.313 | 0.361 | 0.469 | 10 |
| | **Weight (lbs.)** | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| **4** | **Ave Stress Ratio** | 0.305 | 0.303 | 0.341 | 0.358 | 0.367 | 0.416 | 0.540 | 15 |
| | **Weight (lbs.)** | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| **4.5** | **Ave Stress Ratio** | 0.373 | 0.367 | 0.408 | 0.423 | 0.431 | 0.476 | 0.613 | 15 |
| | **Weight (lbs.)** | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| **5** | **Ave Stress Ratio** | 0.447 | 0.436 | 0.479 | 0.491 | 0.497 | 0.538 | 0.688 | 15 |
| | **Weight (lbs.)** | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| **7.5** | **Ave Stress Ratio** | 0.884 | 0.817 | 0.817 | 0.813 | 0.847 | 0.874 | 1.069 | 15 |
| | **Weight (lbs.)** | 18940 | 16643 | N.C | 18060 | N.C | N.C | N.C | |
| **10** | **Ave Stress Ratio** | 1.418 | 1.260 | 1.233 | 1.193 | 1.224 | 1.248 | 1.461 | 15 |
| | **Weight (lbs.)** | 35883 | 33980 | N.C | 35188 | N.C | N.C | N.C | |

**Table 7-Optimum Roof Slope for Santa Barbara, CA**

| L/H | Out put | θ = 10 | θ = 15 | θ = 20 | θ = 25 | θ = 30 | θ = 35 | θ = 45 | Optimum θ (Degree) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Ave Stress Ratio | 0.047 | 0.048 | 0.051 | 0.051 | 0.052 | 0.055 | 0.065 | 10 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 1.5 | Ave Stress Ratio | 0.065 | 0.066 | 0.069 | 0.070 | 0.071 | 0.077 | 0.092 | 10 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 2 | Ave Stress Ratio | 0.090 | 0.090 | 0.098 | 0.102 | 0.106 | 0.111 | 0.125 | 10 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 2.5 | Ave Stress Ratio | 0.133 | 0.132 | 0.141 | 0.145 | 0.148 | 0.154 | 0.170 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 3 | Ave Stress Ratio | 0.184 | 0.181 | 0.191 | 0.194 | 0.196 | 0.201 | 0.217 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 3.5 | Ave Stress Ratio | 0.243 | 0.230 | 0.247 | 0.248 | 0.247 | 0.252 | 0.268 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 4 | Ave Stress Ratio | 0.309 | 0.297 | 0.308 | 0.305 | 0.302 | 0.305 | 0.319 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 4.5 | Ave Stress Ratio | 0.382 | 0.364 | 0.374 | 0.366 | 0.360 | 0.363 | 0.372 | 15 |
| | Weight (lbs.) | N.C | N.C | N.C | N.C | N.C | N.C | N.C | |
| 5 | Ave Stress Ratio | 0.460 | 0.434 | 0.442 | 0.431 | 0.419 | 0.418 | 0.426 | 15 |
| | Weight (lbs.) | 8268 | 8254 | N.C | N.C | 8717 | N.C | N.C | |
| 7.5 | Ave Stress Ratio | 0.932 | 0.842 | 0.787 | 0.747 | 0.741 | 0.723 | 0.709 | 15 |
| | Weight (lbs.) | 24246 | 21140 | N.C | N.C | N.C | N.C | 23630 | |
| 10 | Ave Stress Ratio | 1.505 | 1.315 | 1.196 | 1.113 | 1.085 | 1.046 | 1.037 | 15 |
| | Weight (lbs.) | 38696 | 33192 | N.C | N.C | N.C | N.C | 37912 | |

The optimization problem shows that even though the problem is non-linear, it is not highly non-linear with local minima and maxima. Figure 3 shows that for Berlin, NH with a L/H of 1, that the weights follow a gently curving plot. This is well-behaved with no local maxima or minima. The plot suggests that further reducing the roof slope, such as making it a flat roof, would be optimal. However, because flat roofs have significantly differing requirements for roofing materials, the cost spikes up below 10 degrees and it is not the optimal cost.

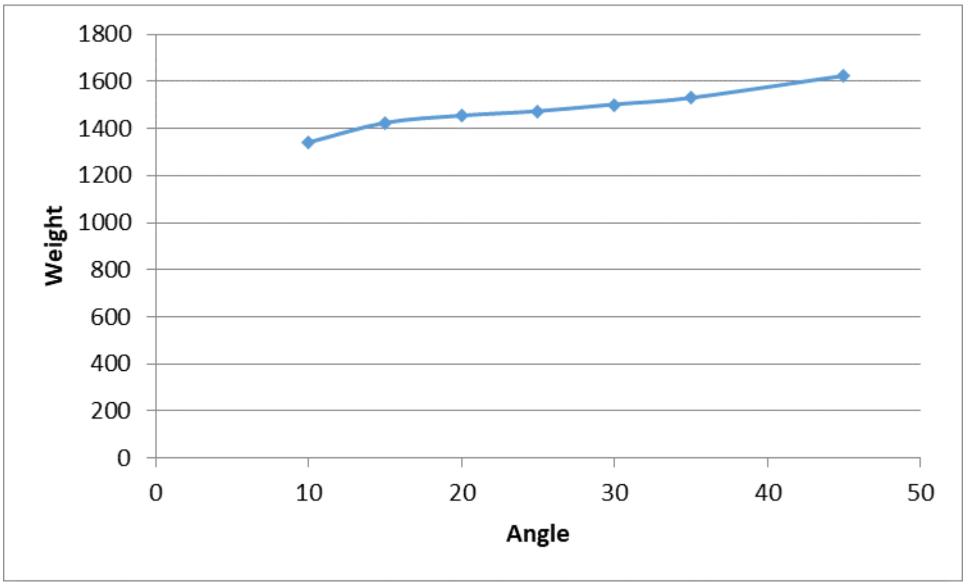


**Figure 3. Nonlinearity of Optimization**

Another observation from Figure 3 is that the stress utilization and weight generally follow the same pattern. This means that the stress utilization is a very good predictor of the weight of the final design. As seen in Table 5, and many other places, the lowest stress utilization generally gives the lowest weight design, regardless of roof angle. An exception is in Table 5 with L/H=10 because the minimum stress is at 45 degrees while the minimum weight is in 30 degrees. The roof angle only makes a few percentages effect difference on the weight (for example between 15 and 20 degree weight difference is about 5 percent. As said above, the lowest roof angle is least weight if it has the lowest stress utilization, but since the roof angle doesn't change the weight much, a shallower roof angle won't control if its stress utilization is more than a few percentage points higher than the next higher angle. The situation for Table 5 with L/H=10 is that the length increases enough between 30 and 45 degrees so that the small difference in stress utilization is overcome.

There are some exceptions to having no local minima as shown in Table 4, for St. Paul, MN with an L/H of 4. Related to this, note that the optimum roof angle is reported as 30.3 degrees even though that was not one of the preselected roof angles to be investigated. The reason for having two local minima for the optimization is that there is a discontinuity in the building code requirements at that point. With a roof slope of 7 to 12 or below, unbalanced snow load needs to be considered, and it is the controlling load case for many gable frame roofs in regions with significant snow. However, on highly sloped roofs, the code doesn't require it. Therefore, at the angle of 30.2 degrees, the discontinuity occurs. In order to legally avoid unbalanced snow, the roof angle would have to be measurably above the break point. It is left to the designer to determine what that margin should be, but it is reported as 30.3 degrees here. Since the unbalanced snow load is no longer required above that roof angle, the stress utilizations drops between 30 and 35 degrees. The actual optimum weight would be lower than the values reported at 30 and 35 degrees. From 30 to 35 degrees the change in member length and weight can be significant, so keeping the weight near to the lower angle is best. Note that in the same table, for an L/H of 3.5, there is also the same phenomenon of a local minimum at 35 degrees, but it is not low enough to override the effect of more weight from longer members at higher angles.

The most important results are for designers to determine which roof angles are optimal. A generalization can be made that for regions with low snow, a low roof angle is optimal. This is defined by looking at the ratio of ground snow load to roof live load. When that ratio is less than 2.5 it is termed low snow for this context. The optimum roof angle starts at 10 degrees for these roofs, and goes to 15 degrees as the roof gets longer. For the cities with low snow, that breakpoint was anywhere from L/H of 2.5 to 4. As snow load gets higher in more northern regions, it becomes more efficient to have a higher sloped roof because the roof snow is shed off the surfaces. For short roofs, with an L/H of less than about 2, the optimum roof angle is 10 to 15 degrees like roofs with low snow. However, as the span increases, the total weight of snow goes up and steeper roofs are preferred. If the ratio of ground snow to roof live load is greater than 2.5, in most cases optimum slope is just greater than

30.2 degrees. The higher the ground snow load, the lower the span will be where this becomes the preferred roof angle.

## IV. CONCLUSION

The optimization of prefabricated non-prismatic steel gable frame roofs doesn't require sophisticated algorithms to solve. Although there are some discontinuities in loading, such as unbalanced snow load, the behavior is understood well. Other non-linearities are not very strong. With the current code and methods, generally in non snow or low snow regions, which is a ratio of ground snow over roof live load is less than 2.5, the the optimum roof slope for steel pre-fabricated gable frames is 10 to 15 degrees. When the ratio of ground snow to roof live load is greater than 2.5, for L/H less than 2 the optimum roof slope is 10 degrees and for higher L/H ratios it is just over 30.2 degrees. It is recommended the roof slope be chosen as 30.3 degrees for in high snow regions for most common gable buildings.

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